Extreme Dependence Structures and the Cross-Section of Expected Stock Returns[☆]

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Abstract

We examine whether investors receive compensation for holding stocks with a strong sensitivity to extreme market downturns. Standard asset pricing models are unable to capture such extreme dependencies because they rely on the linear correlation between a stock and the market as their sole dependence measure. We use copulas to measure lower tail dependence between an individual stock's return and the market return as a proxy for a stock's crash sensitivity. We show that lower tail dependence is indeed an important driver of the cross-sectional variation of expected stock returns. Stocks with strong lower tail dependence deliver positive abnormal returns. This effect cannot be explained by traditional risk factors and is different from the impact of coskewness and downside beta. Our findings are consistent with results from the empirical option pricing literature and support the notion that stock market investors are crash-averse which eventually leads to higher equilibrium returns of crash-sensitive stocks.

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Abstract

We examine whether investors receive compensation for holding stocks with a strong sensitivity to extreme market downturns. Standard asset pricing models are unable to capture such extreme dependencies because they rely on the linear correlation between a stock and the market as their sole dependence measure. We use copulas to measure lower tail dependence between an individual stock's return and the market return as a proxy for a stock's crash sensitivity. We show that lower tail dependence is indeed an important driver of the cross-sectional variation of expected stock returns. Stocks with strong lower tail dependence deliver positive abnormal returns. This effect cannot be explained by traditional risk factors and is different from the impact of coskewness and downside beta. Our findings are consistent with results from the empirical option pricing literature and support the notion that stock market investors are crash-averse which eventually leads to higher equilibrium returns of crash-sensitive stocks.

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1. Introduction

There is strong empirical support from the option pricing literature for the notion that investors are crash-averse: Deep out-of-the-money index puts, i.e., instruments that offer protection against extreme market downturns, have a high implied volatility, i.e., they are relatively expensive.[1] This pattern is typically interpreted as investors being crash-averse or showing signs of 'crash-o-phobia' (Rubinstein (1994), Bates (2008)). More recently, Bollerslev and Todorov (2011) find that most of the aggregate equity risk premium is a compensation for the risk of extreme events. Surprisingly, the potential impact of crash aversion has not caught much attention in the empirical literature on the pricing of common stocks. Our study addresses this issue by investigating the impact of individual stock crash-sensitivity on the cross-section of returns.

Standard asset pricing models since Sharpe (1964) and Lintner (1965) argue that the joint distribution of individual stock returns and the market portfolio return determines the cross-section of expected stock returns. According to the empirical interpretation of the traditional CAPM, a stock's expected return only depends on its linear correlation with the market. In the context of bivariate normal distributions, the linear correlation is the appropriate dependence concept. However, the linear correlation can not characterize the full dependence structure of non-normally distributed random variables such as realized stock returns (Embrechts, McNeil, and Straumann (2002)). Particularly, it can not capture clustering in the tails of the bivariate return distribution between individual securities and the market, which is of foremost importance if investors are crash-averse. We use copula methods based on extreme value theory to determine the crash-sensitivity of a stock. Specifically, we capture stock individual crash-sensitivity based on extreme dependencies between individual stock returns and market returns in the lower left tail of their joint distribution (also called lower tail dependence) and investigate its influence on the cross section of individual stock returns.[2] All else being equal, securities that exhibit strong lower tail dependence (LTD) are unattractive assets for crash-averse investors; such securities tend to realize their lowest payoffs at the time when the market also realizes its lowest payoff, i.e., when investors' wealth

[1] See, e.g., Rubinstein (1994), Jackwerth and Rubinstein (1996), Aït-Sahalia and Lo (2000), Bates (2000), Jackwerth (2000), Rosenberg and Engle (2002), Broadie, Chernov, and Johannes (2009). Garleanu, Pedersen, and Poteshman (2009) show that this effect is driven by high demand for out-of-the-money puts. Bollen and Whaley (2004) also show that buying pressure for index puts increases their price. Grossman and Zhou (1996) suggest an equilibrium model where a group of investors buys portfolio insurance which can explain the high prices of out-of-the-money put options, too.

[2] A positive influence of LTD on returns is expected (but not empirically shown) in Poon, Rockinger, and Tawn (2004): "If tail events are systematic as well, one might expect the extremal dependence between the asset returns and the market factor returns to also command a risk premium." (p. 586).

is already very low (given that one accepts the market as a proxy for investor wealth). Thus, investing in stocks with only weak or no LTD is a kind of insurance against extreme negative portfolio returns (similar to buying out-of-the-money index puts) as these stocks are unlikely to realize their worst returns when the market realizes its worst return. Consequently, investors who are sensitive to extreme downside losses will require a return premium for holding stocks with strong LTD.

Based on daily return data for all US stocks from 1963 to 2009, we calculate LTD coefficients for each stock and year. To do so, we first determine the convex combination of basic parametric copulas that best explains the empirical bivariate distribution of an individual stock's return and the market return. Then, we compute the respective tail dependence coefficients. As expected, we find that average LTD peaks around the 1987 crash as well as during the recent financial crisis starting in 2008.

We then relate individual LTD to contemporaneous returns on an annual basis. In doing so, we follow Lewellen and Nagel (2006) who suggest using short, non-overlapping periods and daily data in asset pricing exercises when risk exposures might be time varying. Our empirical results using portfolio sorts and Fama and MacBeth (1973) regressions on the individual firm level show a strong positive impact of LTD on returns. A portfolio consisting of the 20% stocks with the strongest LTD delivers a contemporaneous return which is 15.71% p.a. higher than that of a portfolio consisting of the 20% stocks with the weakest LTD. In contrast, weak LTD stocks have significantly higher returns than strong LTD stocks during extreme market downturns, i.e., weak LTD stocks indeed offer some protection against extreme market downturns.

The impact of LTD has to be distinguished from the impact of downside beta documented in Ang, Chen, and Xing (2006) as well as from the impact of other higher co-moments. Downside beta focuses on individual securities' exposure to market returns conditional on below-average market returns. It places no particular emphasis on tail events. Thus, downside beta captures general downside risk aversion rather than crash aversion. Lower tail dependence is a fundamentally different concept and captures the dependence in the extreme left tail of return distributions, i.e., it focuses on how individual securities behave during the worst market return realizations. We find a strong impact of LTD even after controlling for the impact of the Ang, Chen, and Xing (2006) downside beta. Adding lower tail dependence as an explanatory variable even drives out the impact of downside beta in a multivariate setting. This suggests that crash aversion has an additional impact on the cross section of returns, which can be distinguished from the impact of downside risk aversion. We can also show that the risk premium associated with LTD cannot be captured by coskewness (Harvey and Siddique (2000)), cokurtosis (Fang and Lai (1997)), idiosyncratic volatility

(Ang, Hodrick, Xing, and Zhang (2006)), or a stock's lottery characteristics (Bali, Cakici, and Whitelaw (2011a)). In our multivariate analysis, we also control for the standard list of firm characteristics that can impact returns including size (Banz (1981)), book-to-market (Basu (1983)), momentum (Jegadeesh and Titman (1993)), and liquidity (Amihud (2002)). Our results from Fama-MacBeth (1973) regressions show that an increase of one standard deviation in LTD is associated with an average return premium of about 5.28% p.a.

Additionally, we conduct several robustness tests to confirm the stability of our results. Results are confirmed if we conduct various risk- and industry-adjustments of a stock's return and change the weighting scheme. In addition, the results are stable over time. We also show that our results are very similar if we do not calibrate the optimal copula structure for each stock and year but instead just estimate tail dependence coefficients based on some fixed ad-hoc copula combinations. This result might be useful for future research, because selecting the right copula combination is computationally costly.

Finally, we find that an investible trading strategy consisting of buying a portfolio of stocks with the strongest past lower tail dependence and selling a portfolio of stocks with the weakest past lower tail dependence over the previous twelve months delivers a significant abnormal return of 4.17% p.a. before trading costs. This result holds after controlling for the Fama and French (1993) factors, momentum, and systematic liquidity risk.

Overall, the main contribution of our study is twofold: (1) We document that extreme dependencies between individual stock and market returns determine expected stock returns. (2) We apply copula methods to capture and use these dependencies in an asset pricing context for the first time. Thus, our study is related to the literature on downside risk- and loss aversion and to the literature on crash aversion as well as to the literature on the application of extreme value theory in finance.

Downside risk aversion is discussed in Roy (1952), who argues that investors display 'safety-first' preferences, and in Markowitz (1959), who suggests using the semi-variance as a measure of risk.[3] Kahneman and Tversky (1979) argue that individuals evaluate outcomes relative to reference points and show that individuals strongly prefer avoiding losses to acquiring gains. Although aversion to losses and downside risk aversion is discussed extensively in the literature, only a few papers investigate the effect of loss- or disappointment aversion on expected asset returns. Shumway (1997) develops and tests an asset pricing model with

[3] Many subsequent contributions analyze the impact of higher co-moments on expected returns. Extensions of the basic CAPM that allow for preferences for skewness and lower partial moments of security and market returns are developed by Kraus and Litzenberger (1976) and Bawa and Lindenberg (1977). Kraus and Litzenberger (1976), Friend and Westerfield (1980), and Harvey and Siddique (2000) document that investors dislike negative coskewness of stock returns with the market return. Fang and Lai (1997) and Dittmar (2002) show that stocks with high cokurtosis earn high average returns.

loss averse investors who demand an additional premium for risk associated with negative market returns.[4] However, these papers as well as the study by Ang, Chen, and Xing (2006) are concerned with general downside risk aversion rather than crash aversion.

Crash aversion has still caught relatively little attention in the asset pricing literature on common stocks. A small number of recent papers examine the time-series relationship between tail risk and *aggregate* stock market returns (e.g., Bali, Demirtas, and Levy (2009), Kelly (2011), Bollerslev and Todorov (2011)). They find that proxies for tail risk can predict aggregate market returns. The only other papers we are aware of that investigate whether tail risk exposure has an impact on the cross-section of individual stock returns are Kelly (2011), Cholette and Lu (2011), and Bali, Cakici, and Whitelaw (2011b). The latter paper extends the Bawa and Lindenberg (1977) mean-lower partial moment CAPM and looks at co-lower partial moments of stocks with the market conditional on the individual stock being below a specified threshold. Conditioning on the individual stock (rather than the market) being below a specific threshold is motivated by the argument that many investors are not diversified. They find that stocks with such high co-lower partial moments outperform stocks with low co-lower partial moments. Kelly (2011) predicts aggregate tail risk based on individual extreme events by applying the tail risk estimator of Hill (1975) to the cross-section of all daily stock returns in a given month.[5] He finds that this measure predicts aggregate market returns and documents that a long-short portfolio that is based on individual stocks' exposure to an aggregate tail risk factor and that hedges against tail events delivers negative returns of 23 to 72 bp per month. However, these results are based on a non-investable trading strategy because he estimates the aggregate tail risk factor and individual return exposures using the whole sample of data and finds no significant results if he does an out of sample analysis. Our paper differs from his approach by capturing tail dependence directly based on the joint distribution of each individual stock return and the market return. Thus, our approach offers the advantage that we only need a time series of an individual stock's return and the market return (rather than the whole cross section of all individual stock returns at each point in time) to estimate tail dependence. Furthermore, in contrast to Kelly (2011), we find some evidence that tail dependence predicts returns also in an investable trading strategy.

We also contribute to the finance literature methodologically. Despite its long history in

[4]Barberis and Huang (2001), in one of their model variants, study equilibrium firm-level stock returns when investors are loss averse over the fluctuations of the individual stocks in their portfolio. They predict a large value premium in the cross-section. Other models with loss averse investors include Benartzi and Thaler (1995) and Barberis, Huang, and Santos (2001), while Ang, Bekaert, and Liu (2005) model investors with disappointment aversion.

[5]A similar approach is followed in Cholette and Lu (2011).

statistics, multivariate extreme value theory has been applied to the analysis of financial markets only recently.[6] It is used to describe dependence patterns across different markets and assets (Longin and Solnik (2001)). However, to the best of our knowledge, ours is the first paper to investigate extreme dependence structures in the bivariate distribution of the returns of individual stocks and the market based on copulas. Our application details how to fit flexible combinations of basic parametric copulas to this bivariate distribution and how to compute tail dependence coefficients based on them. The copula approach has the advantage that extreme dependence is not estimated based on a small number of observations in the tail exclusively, but that information from the whole joint distribution can be used.

The rest of this paper is organized as follows. Section 2 gives a short overview of copula theory and presents the estimation procedure for tail dependence coefficients. Section 3 demonstrates that stocks with strong lower tail dependence contemporaneously have high average returns. In Section 4, we examine the persistence of tail dependence and evaluate a trading strategy based on lower tail dependence. Section 5 concludes.

2. Copulas and Tail Dependence Coefficients

As copula concepts are not yet regularly used in standard asset pricing applications, we will first give a short intuitive introduction into the concept (Section 2.1), explain how we compute measures of extreme or tail dependence based on copulas (Section 2.2), and describe the development of aggregate tail dependence over time (Section 2.3).[7]

2.1. Copulas

Most of the standard empirical asset pricing literature focuses on risk factors based on linear correlation coefficients. However, this measure of stochastic dependence is not typically able to completely characterize the dependence structure of non-normally distributed random variables (Embrechts, McNeil, and Straumann (2002)). It is now widely recognized that many financial time series, including stock returns, are non-normally distributed.[8] For example, they are often characterized by leptokurtosis. This is problematic because when we are dealing with a fat-tailed bivariate distribution $F(x_1, x_2)$ of two random variables X_1 and

[6] Longin and Solnik (2001) use extreme value theory to model the bivariate return distributions between different international equity markets. Focusing on risk management applications, Ané and Kharoubi (2003) propose modeling the dependence structure of international stock index returns via parametric copulas and derive their tail dependence behavior. Poon, Rockinger, and Tawn (2004) present a general framework for identifying joint-tail distributions based on multivariate extreme value theory. They argue that the use of traditional dependence measures could lead to inaccurate portfolio risk assessment.

[7] A more precise technical, but still accessible, treatment of copula concepts is contained in Nelsen (2006).

[8] For some early evidence, see Fama (1965). A discussion of properties of empirical asset returns can be found in Cont (2001).

X_2, the linear correlation fails to capture the dependence structure in the extreme lower left and upper right tail. As an example, consider the following illustrations of 2000 simulated bivariate realizations based on different dependence structures between (X_1, X_2) shown in Figure A.1.[9]

[Insert Figure A.1 about here]

In all models, X_1 and X_2 have standard normal marginal distributions and a linear correlation of 0.8, but other aspects of the dependence structure are clearly different. For comparison, in Panel A we first show an example where we did not allow for clustering in either tail of the distribution. Panels B to D show examples of increased dependence in the upper right tail, in the lower left tail, and symmetric increased dependence in both tails. Still, all of these bivariate distributions are characterized by a linear correlation coefficient of 0.8. These examples show that it is not always possible to describe the dependence structure by the linear correlation alone.

Copulas offer an elegant way to describe the complete dependence structure between random variables. Every distribution function of two random variables X_1 and X_2 (e.g., an individual asset return and the market return) implicitly contains both a description of the marginal distribution functions $F_1(x_1)$ and $F_2(x_2)$ and their dependence structure. The copula approach allows us to isolate the description of the dependence structure from the univariate marginal distributions of the bivariate distribution. Sklar (1959) shows that all bivariate distribution functions $F(x_1, x_2)$ can be completely described based on the univariate marginal distributions and a copula C: $[0,1]^2 \to [0,1]$. Sklar's Theorem explicitly states that all bivariate distributions can be decomposed into copulas and that the marginal distributions and bivariate distributions can be constructed by combining the univariate marginal distributions using copulas (McNeil, Frey, and Embrechts (2005)). Formally, Sklar's Theorem states:

Theorem 1 (Sklar 1959)**.** *Let F be a bivariate distribution function with margins F_1 and F_2. Then there exists a copula $C : [0,1]^2 \mapsto [0,1]$ such that, for all x_1, x_2 in $\overline{\mathbb{R}} = [-\infty, \infty]$,*

$$F(x_1, x_2) = C(F_1(x_1), F_2(x_2)). \tag{1}$$

If the margins are continuous, then C is unique. Conversely, if C is a copula and F_1 and F_2 are univariate distribution functions, then the function F defined in (1) is a bivariate

[9]The realizations plotted in Figure A.1 are based on simulations of different popular copula functions. The statistical models used to simulate these realizations are the Gauss-copula (Panel A), the Gumbel-copula (Panel B), the Clayton-copula (Panel C), and the Student t-copula (Panel D), all as defined in Table A.1.

distribution function with margins F_1 and F_2.

There are many different parametric copulas C that can model different tail dependence structures. In this study, we use combinations of simple basic parametric copulas that either show no tail dependence (the Gauss-, the Frank-, the FGM-, and the Plackett-copula), lower tail dependence (the Clayton-, the Rotated Gumbel-, the Rotated Joe-, and the Rotated Galambos-copula), or upper tail dependence (the Gumbel-, the Joe-, the Galambos-, and the Rotated Clayton-copula). By using a convex combination of one copula from each class, we allow for maximum flexibility in modeling dependence structures (see Section 2.2).

We will later use copulas to estimate coefficients of upper and lower tail dependence (Sibuya (1960)), i.e., measures of the strength of dependence in the tails of a bivariate distribution.

2.2. Computation of Tail Dependence Coefficients

Intuitively, the lower (upper) tail dependence coefficient between two variables reflects the probability that a realization of one random variable is in the extreme lower (upper) tail of its distribution conditional on the realization of the other random variable also being in the extreme lower (upper) tail of its distribution. Formally, lower tail dependence LTD is defined as

$$\text{LTD} := \text{LTD}(X_1, X_2) := \lim_{u \to 0+} P(X_1 \leq F_1^{-1}(u) | X_2 \leq F_2^{-1}(u)), \tag{2}$$

where $u \in (0, 1)$ is the argument of the distribution function, i.e., $\lim_{u \to 0+}$ indicates the limit if we approach the left-tail of the distribution from above. Analogously, we can define the upper tail dependence coefficient UTD as:

$$\text{UTD} := \text{UTD}(X_1, X_2) := \lim_{u \to 1-} P(X_1 > F_1^{-1}(u) | X_2 > F_2^{-1}(u)). \tag{3}$$

If LTD (UTD) is equal to zero, the two variables are asymptotically independent in the lower (upper) tail. Simple expressions for LTD and UTD in terms of the copula C of the bivariate distribution can be derived based on

$$\text{LTD} = \lim_{u \to 0+} \frac{C(u, u)}{u} \tag{4}$$

and

$$\text{UTD} = \lim_{u \to 1-} \frac{1 - 2u + C(u, u)}{1 - u}, \tag{5}$$

if F_1 and F_2 are continuous (McNeil, Frey, and Embrechts (2005)). The coefficients of tail dependence have closed form solutions for many parametric copulas. Table A.1 shows the formulas for the computation of LTD and UTD of the basic copula families used in this study.

[Insert Table A.1 about here]

These basic copulas do not allow us to model upper and lower tail dependence simultaneously. However, flexible copula structures can be obtained by constructing convex combinations of these basic copulas. Tawn (1988) shows that every convex combination of existing copula functions is again a copula. Thus, if $C_1(u_1, u_2)$, $C_2(u_1, u_2)$, ..., $C_n(u_1, u_2)$ are bivariate copula functions, then

$$C(u_1, u_2) = w_1 \cdot C_1(u_1, u_2) + w_2 \cdot C_2(u_1, u_2) + \ldots + w_n \cdot C_n(u_1, u_2)$$

is again a copula for $w_i \geq 0$ and $\sum_{i=1}^{n} w_i = 1$.

To allow for the maximum possible flexibility, we consider all 64 possible convex combinations of the afore mentioned basic copulas from Table A.1 that each consist of one copula that allows for asymptotic dependence in the lower tail, C_{LTD}, one copula that is asymptotically independent, C_{NTD}, and one copula that allows for asymptotic dependence in the upper tail, C_{UTD}:

$$\begin{aligned} C(u_1, u_2, \Theta) = {} & w_1 \cdot C_{\text{LTD}}(u_1, u_2; \theta_1) \\ & + w_2 \cdot C_{\text{NTD}}(u_1, u_2; \theta_2) + (1 - w_1 - w_2) \cdot C_{\text{UTD}}(u_1, u_2; \theta_3), \end{aligned} \tag{6}$$

where Θ denotes the set of the basic copula parameters θ_i, $i = 1, 2, 3$ and the weights w_1 and w_2. These convex combinations are similar to other copulas such as the BB1 to BB7 copulas suggested in Joe (1997), but offer more flexibility. Particularly, as our convex combinations also contain one copula that is asymptotically independent, ours is an extremely flexible and efficient way to model dependence structures.

Our estimation approach for the upper and lower tail dependence coefficients follows a three-step procedure. First, based on daily return data for the market and each stock, we estimate a set of copula parameters Θ_j for $j = 1, \ldots, 64$ different copulas $C_j(\cdot, \cdot; \Theta_j)$ between an individual stock return r_i and the market return r_m for each year.[10] Each of these convex combinations requires the estimation of five parameters: one parameter θ_i ($i = 1, 2, 3$) for each of the three basic copulas and two parameters for the weights w_1 and w_2. The copula

[10] In computing the market return r_m we exclude stock i, so the market return r_m is slightly different for each stock's time series regression. This removes potential endogeneity problems when calculating LTD- and UTD-coefficients for each stock.

parameters Θ_j are estimated via the canonical maximum likelihood procedure of Genest, Ghoudi, and Rivest (1995).

Second, we follow Ané and Kharoubi (2003) and select the appropriate parametric copula $C^*(\cdot,\cdot;\Theta^*)$ by minimizing the distance between the different estimated parametric copulas $C_j(\cdot,\cdot;\widehat{\Theta}_j)$ and the empirical copula $\widehat{C}$ based on the Integrated Anderson-Darling distance.[11] The result of this step is summarized in Table A.2, in which we present the absolute and percentage frequency by which each of the possible 64 combinations is chosen.

[Insert Table A.2 about here]

All combinations are chosen regularly and no specific copula clearly dominates. The three copula combinations that are most often selected are the Rotated-Joe/F-G-M/Joe-copula (3.29%), the Rotated-Galambos/F-G-M/Joe-copula (3.08%), and the Rotated-Gumbel/F-G-M/Joe-copula (2.65%).

Third, we compute the tail dependence coefficients LTD and UTD implied by the estimated parameters Θ^* of the selected copula $C^*(\cdot,\cdot;\Theta^*)$ based on the respective formulas for LTD and UTD from Table A.1. The lower and upper tail dependence coefficient of the convex combination are calculated as the weighted sum of the LTD and UTD coefficients from the individual copulas, respectively, where the weights from (6) are used, i.e., $\mathrm{LTD}^* = w_1^* \cdot \mathrm{LTD}(\theta_1^*)$ and $\mathrm{UTD}^* = (1 - w_1^* - w_2^*) \cdot \mathrm{UTD}(\theta_3^*)$. As this procedure is repeated for each stock and year, we end up with a panel of tail dependence coefficients $\mathrm{LTD}^*_{i,t}$ and $\mathrm{UTD}^*_{i,t}$ at the year-firm level. For a more detailed description of the estimation and selection method, we refer the reader to the Appendix.

2.3. Data and the Evolution of Aggregate Tail Dependence

Our sample consists of all common stocks (CRSP share codes 10 and 11) from CRSP trading on the NYSE/AMEX between January 1, 1963 through December 31, 2009. Copulas and tail dependence coefficients are estimated for each firm and each year separately. To estimate the yearly tail dependence coefficients, we use daily data for all days on which the stock's price at the end of the previous trading day was at least \$2. We retain all stocks that have at least 100 valid daily return observations per year. Overall, there are $96,676$ firm-year observations. The number of firms in each year over our sample period ranges from $1,489$ to $2,440$.

To get a first idea about the characteristics of tail dependence, we investigate the time series of aggregate LTD and aggregate UTD. We define aggregate LTD of the market, $\mathrm{LTD}_{m,t}$, as

[11] Results are very similar whether we select the copula based on the Kolmogorov-Smirnov distance or the estimated log-likelihood value. For a detailed description of the selection procedure, see the Appendix.

the yearly cross-sectional, equally-weighted, average of $\mathrm{LTD}_{i,t}$ over all stocks i in our sample. Analogously, we define aggregate UTD of the market, $\mathrm{UTD}_{m,t}$, as the yearly cross-sectional, equally-weighted, average of $\mathrm{UTD}_{i,t}$. In Figure A.2, we plot the time series of $\mathrm{LTD}_{m,t}$ and $\mathrm{UTD}_{m,t}$.

[Insert Figure A.2 about here]

There is no particular time trend in $\mathrm{LTD}_{m,t}$ and $\mathrm{UTD}_{m,t}$.[12] The time series of $\mathrm{LTD}_{m,t}$ and $\mathrm{UTD}_{m,t}$ are positively correlated with a linear correlation coefficient of 0.29. However, the graph does exhibit occasional spikes in $\mathrm{LTD}_{m,t}$ that roughly correspond to worldwide financial crises. The highest value in $\mathrm{LTD}_{m,t}$ corresponds to 1987, the year of Black Monday with the largest one-day percentage decline in US stock market history. Another spike in market LTD occurs during the years 2007 through 2009, the years of the recent worldwide financial crisis. This suggests that $\mathrm{LTD}_{m,t}$ – similar to return correlations – increases in times of financial crises.

Figure A.2 also shows that in most of the years of our sample $\mathrm{LTD}_{m,t}$ clearly exceeds $\mathrm{UTD}_{m,t}$. We report levels of and differences between $\mathrm{LTD}_{m,t}$ and $\mathrm{UTD}_{m,t}$ for the whole sample and for 5-year subsamples from 1963 to 2009 in Table A.3.

[Insert Table A.3 about here]

Over the whole sample period, $\mathrm{LTD}_{m,t}$ is on average 0.131, which is significantly higher than the average $\mathrm{UTD}_{m,t}$ of 0.083. This also holds true for each five year subperiod we consider. The general tendency for stronger asymptotic dependence in the left tail than in the right tail of the distributions is consistent with the well-documented finding that return correlations generally increase in down markets.[13]

To get a first impression of the characteristics of firms with strong tail dependence, in Table A.3 we also look at the differences in LTD (UTD) between large and small firms. In every 5-year subsample, we sort firms into five size quintiles and investigate the difference in LTD (UTD) between the large firm and the small firm quintiles. We find that LTD (UTD) is significantly higher for large firms than for small firms. This makes intuitive sense, as a market crash has to be reflected in the returns of large stocks.

[12]Performing two augmented Dickey-Fuller tests rejects the null hypothesis that $\mathrm{LTD}_{m,t}$ contains a unit root with a p-value smaller that 2% and that $\mathrm{UTD}_{m,t}$ contains a unit root with a p-value smaller than 1%.

[13]See, e.g., Ang and Chen (2002). Consistent with our results, Jondeau (2010) finds that lower tail dependence between Fama-French portfolios and the market is significantly stronger than upper tail dependence. Increased extreme dependence between international markets in bear markets is also documented in Longin and Solnik (2001) and Poon, Rockinger, and Tawn (2004).

Correlations among yearly individual security upper and lower tail dependence coefficients and other security characteristics are displayed in Table A.4.[14]

[Insert Table A.4 about here]

The correlation between UTD and LTD is relatively moderate at 0.12. The low correlation shows that firms with strong tail dependence in one tail of the distribution do not necessarily exhibit strong tail dependence in the other tail. This finding also justifies our flexible modeling approach for tail dependence which allows for asymmetric tail dependence in the upper and lower tail. Lower tail dependence is closely related to downside beta with a correlation coefficient of 0.49 and to beta with a correlation coefficient of 0.38. There is also a positive (negative) correlation between LTD and size as well as cokurtosis (illiquidity as well as coskewness). We will make sure to carefully take into account the impact of these correlations in our later analysis.

3. Extreme Dependence and Realized Returns

We start our empirical investigation by looking at the contemporaneous relationship between extreme dependence and returns in univariate sorts (Section 3.1) and double-sorts (Section 3.2). In Section 3.3, we conduct Fama and MacBeth (1973) regressions to control for various other potential determinants of returns and in Section 3.4, we investigate whether we get similar results if we use a simplification of the estimation procedure for tail dependence coefficients. Results from a battery of robustness tests are presented in Section 3.5.

In the main part of our empirical analysis in this section we relate realized tail dependence coefficients to portfolio and individual security returns over the same period. This procedure, which closely follows papers like Ang, Chen, and Xing (2006) and Lewellen and Nagel (2006), implicitly assumes that realized returns are on average a good proxy for expected returns and is mainly motivated by the fact that several studies document that risk exposures (like regular beta) are time-varying (see, e.g., Fama and French (1992), and Ang and Chen (2007)). It is likely that extreme dependence is also time-varying and past extreme dependence might not necessarily be a good predictor of future extreme dependence.[15] As advocated by Lewellen and Nagel (2006), we thus use daily return data for non-overlapping intervals of one year. Over each annual period t, we calculate a stock i's LTD-coefficient (as well as other risk

[14] The exact procedure for the calculation of the other variables is described in Sections 3.2 and 3.3.

[15] In unreported tests we find evidence for only limited predictive power of past LTD on current LTD (see also Section 4.1).

measures like regular beta or downside beta).[16] Using an annual horizon trades off two concerns: First, we need a sufficiently large number of observations to get reliable estimates for our tail dependence coefficients. Second, by investigating contemporaneous relationships over relatively short horizons we are able to account for time-varying extreme dependence.

3.1. Univariate Portfolio Sorts

To examine whether stocks with strong lower tail dependence earn a premium, we first look at simple univariate portfolio sorts. For each year t we sort stocks into five quintiles based on their realized LTD in the same year. Panel A of Table A.5 reports the annual equally-weighted average excess return over the risk free rate for these quintile portfolios.[17] We also report differences in average excess returns between quintile portfolio 5 (strong LTD) and quintile portfolio 1 (weak LTD).

[Insert Table A.5 about here]

We find that stocks with strong LTD have significantly higher average returns than stocks with weak LTD. Stocks in the quintile with the lowest (highest) LTD earn an annual average excess return of 3.99% p.a. (19.70% p.a.). The return spread between quintile portfolio 1 and 5 is 15.71% p.a., which is statistically significant at the 1% level. While only univariate in nature, these results are consistent with investors' being crash averse and requiring a premium for holding stocks with strong LTD.

In Panel A, we also report the average LTD-coefficients, the average contemporaneous regular beta, β, the downside beta, β^{-}, of Ang, Chen, and Xing (2006), the average coskewness (all estimated based on daily data), the average size, and the average book-to-market value of the stocks in each quintile portfolio.[18] Results for average LTD coefficients show that there is a significant cross-sectional dispersion in LTD across quintiles. Average LTD in the lowest quintile is 0.01, while it rises to 0.29 for the highest quintile. We also find a strong relationship between LTD and β, β^{-}, and coskewness. The average (downside) beta in the weakest LTD quintile is (0.43) 0.56 and rises monotonically up to (1.54) 1.19 in the strongest LTD quintile. Average coskewness in the weakest LTD quintile is 0.00 and decreases monotonically to -0.17 in the strongest LTD quintile. Thus, strong LTD stocks may earn high

[16] The estimation procedure of the LTD- and UTD-coefficients for each stock is performed according to Section 2.2 and the Appendix.

[17] Results are stable if we use value-weighting instead of equal-weighting, see Section 3.5.

[18] We compute size as the log of market capitalizations and the book-to-market ratio as the fraction of book value (obtained from Compustat) and market capitalization at the beginning of year t. Downside beta and coskewness are defined in (7) and (8).

returns because they have high regular or downside betas, or negative coskewness. There also is a positive (negative) relationship between LTD and size (book-to-market).

Panel B of Table A.5 shows the relationship between realized average excess returns and UTD. The average return difference between quintile portfolios 1 and 5 of -6.02% p.a. is statistically significant but the effect is much smaller than the impact of LTD from Panel A. This result suggests that investors might have some weak preferences for stocks with extreme dependence in the upper tail. However, these preferences seem much weaker than the preference for avoiding extreme downside losses. Thus, in the remainder of this paper we will mainly focus on LTD.

In summary, results from Table A.5 suggest that extreme dependence determines the cross-section of stock returns. Stocks with strong LTD (UTD) earn high (low) average contemporaneous returns. This is consistent with the view that stocks with weak LTD offer protection against extremely low returns in crises periods, while stocks with strong LTD in those periods perform particularly badly.

As a consistency check, we now limit our evaluation to crises periods only in order to check whether stocks with weak LTD really earn higher returns than strong LTD stocks in these special periods. We analyze portfolio returns on the most relevant financial crises days in our sample period. We examine "Black Monday" (October 19, 1987), the Asian Crisis (October 27, 1997), the burst of the dot-com bubble (April 14, 2000), and the recent Lehman crises (October 15, 2008). Results are presented in Table A.6.

[Insert Table A.6 about here]

As expected (and opposite to what we find in the overall sample), the strong LTD portfolio strongly underperforms the weak LTD portfolio in each case. The differences are economically large. The daily return of the weak LTD portfolio is from 4.4% to 9.2% higher than that of the strong LTD portfolio on those specific days. To assess the statistical significance, we also analyze all days on which the excess return of the market over the riskfree rate was less than -5% together. Results are presented in the last column of Table A.6 and show that the weak LTD portfolio outperforms the strong LTD portfolio by nearly 5% on those days. The effect is statistically significant at the 1%-level. These findings show that weak LTD stocks can serve as an insurance for loss-averse investors and might explain why overall they earn lower returns than strong LTD stocks, as documented above.

3.2. Bivariate Portfolio Sorts

Our univariate result of higher average returns of strong LTD stocks could be driven by differences in regular or downside beta, or differences in coskewness across the different tail dependence quintiles. This possibility is also suggested by the results from the correlation Table A.4, where (un-)conditional betas and coskewness are the variables most strongly correlated with our tail dependence coefficients. Thus, we now conduct double sorts based on tail dependence and these variables.

3.2.1. Tail Dependence vs. Beta

We start with dependent portfolio double-sorts on regular beta and LTD. We first form quintile portfolios sorted on beta. Then, within each beta quintile, we sort stocks into five portfolios based on LTD. As above, both LTD and beta are computed over the same one-year horizon for which we examine returns.

[Insert Table A.7 about here]

Table A.7 reports equally-weighted average portfolio excess returns over the risk free rate. In Panel A we show the results of the 25 $\beta \times$ LTD portfolios. In all LTD quintiles, high beta stocks outperform low beta stocks. More importantly, in all beta quintiles, we document a nearly monotonic increase in returns from the weak LTD to the strong LTD portfolio. The return difference between the weakest LTD quintile and the strongest LTD quintile within all beta quintiles is economically large and statistically significant at the one percent level. It ranges from 5.23% p.a. in the lowest beta quintile to 18.29% p.a. in the highest beta quintile. The average spread in excess returns amounts to 10.40% p.a. Hence, regular beta risk cannot account for the reward earned by holding stocks with strong LTD.

3.2.2. Tail Dependence vs. Downside Beta

Besides regular beta, LTD is also related to downside beta. We follow Ang, Chen, and Xing (2006) and define downside beta as

$$\beta^{-} = \frac{\mathrm{COV}(r_i, r_m | r_m < \mu_m)}{\mathrm{VAR}(r_m | r_m < \mu_m)}, \tag{7}$$

where r_i (r_m) is security i's (the market's) excess return, and μ_m is the average market excess return. As above, in a first step we form quintile portfolios sorted on β^{-}. Then, within each β^{-} quintile, we sort stocks into five portfolios based on LTD. Panel B of Table A.7 reports equally-weighted average excess returns of the 25 $\beta^{-} \times$ LTD portfolios.

We find that the returns of the low β^{-} portfolios tend to be smaller than those of the high β^{-} portfolios. Overall, these findings generally confirm the results of Ang, Chen, and Xing (2006). Turning to the impact of LTD, we find that stocks in the weak LTD portfolios have an average (across all downside beta quintiles) excess return of 7.09% p.a., while stocks in the strong LTD portfolios have an average excess return of 15.57% p.a. This spread is significant at the 1% level. Amounting to 8.48% p.a., the spread is also economically large (ranging from 5.16% p.a. to 12.98% p.a. within the individual β^{-} quintiles). We find the strongest effect of LTD on returns in the highest β^{-} quintile, but it is highly statistically significant within each individual β^{-} quintile. Hence, the impact of LTD on returns does not seem to be driven by downside beta.

3.2.3. Tail Dependence vs. Coskewness

Harvey and Siddique (2000) show that lower coskewness is associated with higher expected returns. Thus, the return premium for stocks with strong LTD could be driven by their low coskewness (see Table A.4 and Table A.5) defined as

$$coskew = \frac{E[(r_i - \mu_i)(r_m - \mu_m)^2]}{\sqrt{\mathrm{VAR}(r_i)}\mathrm{VAR}(r_m)}. \tag{8}$$

Thus, to explicitly control for the impact of coskewness, in a first step we form quintile portfolios sorted on coskewness. Then, within each coskewness quintile, we sort stocks into five portfolios based on LTD. Panel C of Table A.7 shows equally-weighted average excess returns of the 25 coskewness $\times$ LTD portfolios.

In most coskewness quintiles, we document a monotonic increase in average returns with LTD. The return difference between the weakest LTD quintile and the strongest LTD quintile ranges from 9.74% p.a. to 13.73% p.a. with an average spread of 12.34% p.a. This spread again is highly significant at the 1% level in each coskewness quintile, which indicates that coskewness risk cannot account for the reward earned by holding stocks with strong LTD. At the same time, we can confirm the negative impact of coskewness on returns documented previously in Harvey and Siddique (2000).

To summarize, based on double sorts we provide strong evidence that the risk associated with LTD is different from risks associated with market beta, downside beta, and coskewness. Double sorts offer the advantage that they also allow us to detect potential non-linearities. However, in double sorts we can only control for one stock characteristic at a time. Thus, we now turn to a multivariate approach that allows us to examine the joint impact of different return and other characteristics of the firm that might have an impact on the cross-section of stock returns.

3.3. Multivariate Evidence

To confirm whether there is a premium for strong LTD in the cross-section of expected stock returns, we now run Fama and MacBeth (1973) regressions of firm returns on firm characteristics on the individual firm level in the period from 1963 - 2009 using non-overlapping data.[19] Table A.8 presents the regression results of excess returns on realized extreme dependence and various combinations of regular and conditional betas as well other control variables.

[Insert Table A.8 about here]

In regression (1), we only include regular market beta as explanatory variable and confirm that it carries a significant positive coefficient. In regression (2), we replace regular beta by downside- and upside beta.[20] We find that the impact of downside beta is highly significant, while the impact of upside beta is not significant. This finding is broadly consistent with the analysis and results in Ang, Chen, and Xing (2006) who find that downside beta is an important determinant of the cross-section of stock returns while upside beta has much less of an impact. More important in our context, regression (3) tests the importance of LTD and UTD when explaining the cross-section of individual stock returns. Both LTD and UTD are significant at the 1% level and carry the expected signs. The magnitude of LTD is nearly twice as large as that of UTD.

In the following regressions, we expand our multivariate model and add various other firm characteristics including size, book-to-market, coskewness, illiquidity, past returns, idiosyncratic volatility, cokurtosis, and a stock's lottery features captured by the maximum daily return over the past year (Bali, Cakici, and Whitelaw (2011a)) as described in more detail below.[21] Ang, Hodrick, Xing, and Zhang (2006) and 2009 find that idiosyncratic volatility has a negative impact on returns. Idiosyncratic volatility is calculated as the standard deviation of CAPM-residuals of daily firm returns in year t.

A strong positive impact of illiquidity on returns is documented in many studies (e.g., Amihud (2002)). To control for the impact of liquidity, we thus add the Amihud Illiquidity Ratio, defined as

$$illiq_{i,t} = \frac{1}{Days_t^i} \cdot \sum_{d=1}^{Days} \frac{|r_{i,d}|}{Vol_{i,d}}, \tag{9}$$

[19] This econometric procedure has the disadvantage that risk factors are estimated less precisely in comparison to using portfolios as test assets. However, Ang, Liu and Schwarz (2010) show analytically and demonstrate empirically that the smaller standard errors of risk factor estimates from creating portfolios does not necessarily lead to smaller standard errors of cross-sectional coefficient estimates. Creating portfolios destroys information by shrinking the dispersion of risk factors and leads to larger standard errors.

[20] Upside betas, β^+, are defined analogously to downside betas as: $\beta^+ = \frac{\mathrm{COV}(r_i, r_m | r_m > \mu_m)}{\mathrm{VAR}(r_m | r_m > \mu_m)}$.

[21] Size and book-to-market are computed as before. As book-to-market ratios can get very large if prices are low, we winsorize all realizations of our independent variables at the 1% and 99% levels in order to avoid outliers driving our results. Our results do not hinge on this winsorization (see Section 3.5).

where $Vol_{i,d}$ is security i's trading volume in dollars on day d and $Days_t^i$ is the number of trading days in year t.[22] Amihud (2002) documents an positive and highly significant effect of $illiq$ on expected returns.

The findings of Fang and Lai (1997) and Dittmar (2002) predict a positive impact of cokurtosis on expected returns. Thus, we also include cokurtosis, defined as

$$cokurt = \frac{E[(r_i - \mu_i)(r_m - \mu_m)^3]}{\sqrt{\mathrm{VAR}(r_i)}\mathrm{VAR}(r_m)^{3/2}}. \tag{10}$$

Finally, some recent papers document that investors have a preference for lottery-like assets. In this context, Bali, Cakici, and Whitelaw (2011a) examine the role of extreme positive daily returns in the cross-sectional pricing of stocks. They find that stocks with the highest maximum daily return over the past one month period have low future returns. Our regressions are conducted on a yearly basis. Thus, we control for the past maximum daily returns over the past one year period, which we denote as max.[23]

Results are presented in columns 4 to 8 of Table A.8. Regression (4) includes a restricted number of explanatory variables. It confirms a standard set of cross-sectional return patterns: Market beta (+), book-to-market (+), coskewness (-), and illiquidity (+) are significant explanatory variables for the cross-section of stock returns. In regression (5) we add our measures of extreme dependence. Results show that the premium for LTD is robust to controlling for the impact of these variables. In regressions (6) to (8), we expand the set of independent variables by including past returns, idiosyncratic volatility, downside- and upside beta, cokurtosis, as well as max. We find that most of these additional variables are not uniformly significant determinants for the cross-section of stock returns in our sample across our regression models, while LTD exhibits the strongest influence of all variables in terms of statistical power in all models (e.g., a t-statistic of 10.01 in the full regression model (8)). The last column illustrates the economic significance of our results. Based on Regression (8), we show the change in contemporaneous returns p.a. if the respective independent variable changes by one standard deviation from its mean. For a one standard deviation increase in LTD, annual returns increase by 5.28%. This is the third largest (after beta and size) impact of all the variables included. Generally, beta, book-to-market, LTD, and illiquidity seem to be the most important and consistently priced factors for the cross-section of stock returns with stable coefficients across all specifications.

[22] To ensure that our results are not driven by extreme outliers, we cap the yearly illiquidity measure at a maximum value of 0.3 (30%) (as in Acharya and Pedersen (2005)).

[23] Our later examination of a trading strategy is conducted on a monthly basis. Thus, we will later include a 'lottery-factor' in that analysis, which matches the time-horizon of Bali, Cakici, and Whitelaw (2011a) more closely (see Section 4.2).

3.4. Simplifying Tail Dependence Coefficient Estimation

The copula selection procedure described in Section 2.2 and the Appendix is computationally costly.[24] We now investigate whether this estimation procedure could be simplified. Instead of selecting the appropriate parametric copula by minimizing the distance between 64 different convex copula combination and the empirical copula, we ex ante choose various fixed convex copula combination. As our fixed copula combinations, we consider the Rotated Joe/F-G-M/Joe (3-D-I)-, the Rotated Galambos/F-G-M/Joe (4-D-I)-, the Rotated Gumbel/F-G-M/Joe (2-D-I)-, the Rotated Gumbel/Frank/Gumbel (2-B-II)-, the Rotated Galambos/Frank/Gumbel (4-B-II)-, and the Rotated Galambos/Frank/Galambos (4-B-III)-copula out of Table A.2. We choose the copula combinations (3-D-I), (4-D-I), and (2-D-I) because they are the copulas most often selected in the estimation procedure in Table A.2. In contrast, the remaining three copula combinations are the copulas least often selected in the estimation procedure. We perform Fama-Macbeth (1973) regression of excess returns on LTD (estimated based on the fixed copula combinations) as well as the full set of control variables (as in Regression (8) of Table A.8). Results on the coefficient estimates for the influence of LTD are displayed in the first three columns of Table A.9.

[Insert Table A.9 about here]

We find that LTD is a highly significant explanatory factor for the cross-section of expected stocks returns independent of the specified convex copula combination. The magnitude of the coefficients and the significance levels of the control variables (not shown in the table) also remain stable across the different specifications. These results document that our main findings are not driven by the tail dependence coefficient estimation procedure. The estimation procedure, which is computationally intensive, can be dramatically simplified by just picking a reasonable convex copula combination (based on the copulas in Table A.1). This might be a helpful result for researchers working on the impact of tail dependence in similar settings.

3.5. Robustness Checks

In this section we conduct a battery of robustness tests to analyze whether our main results from above are stable. We examine the influence of return adjustments (Section 3.5.1) and the weighting scheme in the portfolio sorts (Section 3.5.2), the temporal stability of our

[24]The selection of the optimal copula combination and the estimation of the tail dependence coefficients on a grid cluster takes about 20 seconds per stock and year. This amounts to an entrire estimation time of well above 500 hours.

results (Section 3.5.3), and variations of the regression setup employed in the multivariate tests (Section 3.5.4).[25]

3.5.1. Industry-, DGTW (97)-, and Risk-Adjusted Returns

To check whether any of our findings are driven by industry effects, we repeat our multivariate regressions with the full set of controls but use industry-adjusted returns as dependent variable. Results on the estimates for the impact of LTD are presented in columns 4 to 6 of Table A.9.

In lines 1 and 2, we use the Fama-French 12 and 48 industry classification, and in lines 3 to 6 we use the SIC 2-, 3-, and 4-digit industry classification to define industries. In all cases, the coefficient for the impact of LTD is statistically significant at the one percent level and similar in magnitude to the findings from above. These findings show that our results are not just driven by industry effects.

Additionally, instead of controlling for the impact of stock characteristics by including them as explanatory variables, we now also adjust the return of each stock by subtracting the return of its corresponding Daniel, Grinblatt, Titman, and Wermers (1997) characteristic benchmark (DGTW).[26] Results are presented in the last line of Table A.9. Again, our result of a strongly positive impact of LTD on expected returns remains unaffected.

Finally, we present results from univariate sorts of individual risk-adjusted stock returns. To risk-adjust stock return, for each stock and year we run a one-factor model adjusting returns for their exposure to the market factor, a Fama and French (1993) three-factor model that additionally corrects for the exposure to the size as well as the book-to-market factor, and a Carhart (1997) four-factor model, that additionally controls for the momentum factor. In Table A.10 we report the equal weighted alphas from these three models for each LTD quintile.[27]

[Insert Table A.10 about here]

We find that all alphas are monotonically increasing from the weak LTD portfolio to the strong LTD portfolio. The one-factor CAPM alpha among the weak LTD stocks is on average 1.33% p.a., while it is 12.59% among the strong LTD stocks; the difference amounts to more than 11% p.a. and is statistically significant at the 1%-level. For the three- and four-factor

[25]Besides the robustness tests described here, we also use weekly data instead of daily data and a longer estimation horizon to determine our tail dependence coefficients. Our results (not reported) are similar if we use a longer estimation horizon of 2 years, 3 years, or 5 years instead of 12 months.

[26]The DGTW benchmarks are available via `http://www.smith.umd.edu/faculty/rwermers/ftpsite/Dgtw/coverpage.htm`.

[27]Results (not reported) are stable if we analyze value-weighted alphas.

alphas, we find that the difference between weak LTD and strong LTD stocks amounts to 10.10% p.a. and 8.19% p.a., respectively.[28]

3.5.2. Weighting Scheme

So far, our sorts and double sorts focus on equal weighted portfolios and results thus could be influenced by overweighting the importance of small stocks.[29] Therefore, we now examine annual value-weighted average excess returns of the same univariate as well as bivariate sorts as in Sections 3.1 and 3.2. Results are presented in Table A.11.

[Insert Table A.11 about here]

Univariate value-weighted results for LTD are presented in Panel A. Consistent with our previous results, we find that stocks in the quintile with the weakest (strongest) LTD earn an annual average excess return of -1.03% p.a. (9.45% p.a.). The spread in average excess returns between quintile portfolio 1 and 5 is 10.48% p.a., which is statistically significant at the 1% level.

In Panels B to D of Table A.11 we perform value-weighted double-sorts on LTD and beta, downside beta, and coskewness, respectively. As with equally-weighted portfolios, we document an increase in average excess returns in each beta quintile (Panel B). The return spread between strong and weak LTD stocks ranges from 2.75% p.a. for the lowest beta quintile up to 16.22% p.a. in the highest beta quintile. The return spread within all beta quintiles except for the lowest beta quintile is significant at the 1% level. Stocks in the quintile with the weakest (strongest) LTD earn an annual average excess return over all beta quintiles of 0.97% p.a. (9.34% p.a.). We find similar patterns for the double sorts based on LTD and downside beta (Panel C) as well as double sorts based on LTD and coskewness (Panel D).

In unreported tests, we also perform value-weighted univariate sorts based on UTD and double-sorts based on UTD and the same variables as above. In the univariate case, we find a relatively small spread between the extreme UTD quintiles of -2.23% p.a., which is only marginally significant at the 10% level. In the double sorts, in three of the beta quintiles we cannot document a significant decrease in average excess returns with UTD at all, while there

[28]We do the same analysis based on UTD. Consistent with our earlier results, we find that weak UTD stocks outperform strong UTD stocks. However, the effect is much smaller in absolute terms than for our LTD sorts. For example, the four factor alpha difference amounts to about -2.87% p.a. and is significant at the 5% level only.

[29]Similar concerns can also be raised with respect to the regression results because the regression evidence presented in Table A.8 is essentially also based on equal weighting as each observations enters the cross-sectional Fama-MacBeth (1973) regressions with the same weight.

is some significant spread between strong and weak UTD stocks within the two highest beta quintiles. However, the overall impact of UTD on returns is weak and not stable. Similarly weak results for the impact of UTD are also found in the other double sorts.

Overall, using value weighted portfolios rather than equal weighting does not change our main finding of significantly higher returns of stocks with strong LTD. This shows that our results are not driven by extreme returns of a handful of very small firms.

3.5.3. Temporal Stability

In this section, we explore the temporal stability of our main result. As a first check, we reproduce the results of the univariate sorts from Table A.5 for the first half of our sample 1963 to 1986 and for the second half of our sample 1987 to 2009 separately. Thus, we can check whether the results differ prior to the crash of 1987 and after. In the option pricing literature it is sometimes argued that investors became crash-o-phobic after the experience of the 1987 crash (Rubinstein (1994)). If investors were not crash-averse prior to 1987, we should not see any compensation for LTD in the earlier subperiod. Results are presented in Panel A of Table A.12.

[Insert Table A.12 about here]

We document a return spread between stocks with strong LTD and stocks with weak LTD of 16.24% p.a. in the period 1963 to 1986. Results for the later subperiod from 1987 to 2009 show that the return spread is of similar magnitude at 15.37% p.a. In both cases, the magnitude of the spread is similar to that from the complete sample and significant at the 1% level. This shows that investors on the stock market got compensated for bearing stocks with strong LTD prior to 1987 and suggests that investors on the stock market were crash averse even prior to the crash of 1987.[30] In Panel B, we report results from the same double sorts as in Table A.7 separately for the pre- and post-1987 period. We only report the average return across the beta, downside-beta, and coskewness, respectively, portfolios for the 5 LTD portfolios as well as the difference portfolio. While the dependent sorts based on beta and LTD as well as those based on downside beta and LTD show a more pronounced return spread in the later period (8.45% vs. 12.45% and 6.77% vs. 10.26%), return spreads from dependent sorts based on coskewness and LTD are very similar in the two subperiods (12.71% and 11.95%).

[30]While seemingly contradictory to some of the empirical option pricing literature it should be noted that studies that find no strong 'crash-fear' effect prior to 1987 typically rely on very short pre-87 samples due to the lack of option data availability in this period.

Finally, we look at the results from our multivariate regressions with the full set of explanatory variables for the 1963-1972, 1973-1981, 1982-1990, 1991-1999, and 2000-2009 subperiods. Results of the respective Fama and MacBeth (1973) regressions are presented in the first five columns in Panel C of Table A.12. Even within these relatively short subsamples, we always find a coefficient for the impact of LTD that is of a magnitude similar to that in the full sample. It ranges from 0.358 in the last subperiod of our sample up to 0.627 in the fourth subperiod. It is significant at the 5% level in the first subperiod and significant at the 1% level in all other subperiods. LTD is the most consistently priced factor of all explanatory variables across the subperiods. We also look at subsamples based on up- and down market returns. Results are presented in the last two columns. As expected, the effect of LTD on the cross-section of stock returns is stronger in times of up markets (market return > 0) than in down markets (market return < 0). However, the effect is significant at the 1% level in both cases. This confirms that the strong positive cross-sectional impact of LTD on stock returns is remarkably stable over time and holds in up and down markets.

3.5.4. Regression Methods and Further Stability Checks

Our previous multivariate regression evidence relies on Fama and MacBeth (1973) regressions with winsorized independent variables. We now perform several variations of this basic regression approach on the full set of independent variables for the full time period from $1963 - 2009$. Results are presented in Table A.13.

[Insert Table A.13 about here]

Regression (1) performs a Fama-Macbeth-Regression, but we do not winsorize the independent variables. In regression (2) we perform a pooled OLS regression with time-fixed effects and standard errors clustered by stock. Regression (3) is identical, but we cluster standard errors by the Fama-French 12 industry classification.[31] Regression (4) performs a panel data regression with firm fixed effects. Finally, in regression (5) we regress excess returns on the independent variables via a random-effect panel data regression.

We document that LTD is a highly significant explanatory factor for the cross-section of expected stocks returns independent of the specific regression setup. The point estimate for the influence of LTD is about 0.45 and very similar across regressions. Hence, our results are not driven by the specific regression technique or by a certain dependence structure of the error terms.

[31]Results are virtually unchanged whether we cluster by Fama-French 48 or SIC industries.

In unreported tests, we also examine the impact of LTD within subsamples consisting of firms from one Fama-French 12 industry at a time. The effect is significant within most individual industry subsamples and is strongest in 'Business Equipment', 'Whole Sales, Retail, Services', and 'Manufacturing'.

4. Persistence of Extreme Dependence and Trading Strategy

The previous section documents that LTD has a strong impact on the cross section of contemporaneous returns. We now examine whether tail dependence is persistent over time (Section 4.1) and whether it also predicts cross-sectional return differences. Although not the focus of our paper, we thereby want to check whether a profitable trading strategy could be implemented solely based on past information on tail dependence (Section 4.2).

4.1. Persistence of Extreme Dependence

We first examine whether LTD is time-varying. To do so, we compute the average LTD of the stocks in the LTD quintile portfolios over time. Firms are sorted into quintiles based on their realized LTD in year $t = 1$. Panel A of Figure A.3 displays the evolution of the average equal weighted LTD of these portfolios over the following four years $t = 2$ to $t = 5$.

[Insert Figure A.3 about here]

Visual inspection clearly shows that the stocks in the strong LTD portfolio also have stronger LTD than the stocks from the weak LTD portfolio in the following years. However, the difference quickly shrinks considerably. In unreported results from a regression analysis, we find evidence for limited predictive power of past LTD on current LTD. Regressing past LTD on current LTD in a univariate model delivers a coefficient estimate of 0.248 with a high degree of statistical significance (t-stat: 12.56), but a relatively low R^2 of about 7%. The patterns documented indicate that there is some predictability in extreme dependence, but the sharply decreasing differences shown in Figure A.3 suggest that many stocks are exposed to time-varying tail risk.[32]

4.2. Past Extreme Dependence Risk and Future Returns

We now examine whether it is possible to generate abnormal returns based on prior information about past extreme dependence. Our strategy consists of going long in stocks with

[32] This latter finding justifies our previous approach of relating returns to contemporaneous tail dependence realizations over relatively short horizons (as suggested for time-varying risk exposures in Lewellen and Nagel (2006)).

strong past LTD and going short in stocks with weak past LTD with monthly rebalancing. Specifically, we sort stocks into five quintile portfolios at the beginning of each month based on past LTD estimated over the previous twelve months. Then, we examine equally-weighted returns of these portfolios over the next month and calculate the return difference between the strongest and the weakest LTD portfolio. The sample period is from January 1963 to December 2009, with our first year risk measurement period ending in December 1963 for the first portfolio formation in January 1964.

Our empirical setup requires the estimation of LTD-coefficients for each stock for 552 overlapping 12 month periods. Thus, to reduce the computational effort, we rely on the results from Section 3.4 and simplify the estimation procedure for LTD in selecting the Rotated Joe/F-G-M/Joe (3-D-I)-copula as our fixed copula convex combination for all stocks and periods.[33]

Table A.14 reports the monthly average excess return over the risk free rate for all quintile portfolios based on past LTD. We also report t-statistics of return differences between quintile portfolio 5 (strong LTD) and quintile portfolio 1 (weak LTD).

[Insert Table A.14 about here]

Our findings show that stocks in the strongest past LTD quintile earn an average equally-weighted excess return over the risk free rate of 0.850% per month, while the stocks in the weakest past LTD quintile earn 0.509% per month. Thus, our trading strategy of investing in strong LTD stocks and shorting weak LTD stocks delivers an economically significant future return of 0.341% per month, which translates into an average return premium of 4.17% p.a. The spread in average excess returns between quintile portfolios 5 and 1 is statistically significant at the 1% level.[34]

To check whether exposures to systematic risk factors drive our finding, we regress the monthly return time series of the LTD quintile portfolios as well as the strong and weak difference portfolio on the monthly excess market return and other systematic risk factors. The alphas from these regressions are shown in the last three columns of Table A.14. Results from the CAPM one-factor regressions show that a part of the premium from our trading strategy is indeed due to high sensitivity to current market beta. However, the market factor can only explain a small portion of the return premium. The trading strategy still delivers a monthly alpha of 0.233% (significant at the 5% significance level) after controlling for market beta. When taking into account the size factor (SMB) and the book-to-market

[33] Results based on other ad-hoc chosen copula combinations are very similar.

[34] In unreported tests, we also investigate a trading strategy based on UTD. We find no significant return spreads based on such a strategy.

factor (HML), the alpha of our strategy increases to 0.434% per month. The returns of our trading strategy load significantly negatively on both factors. In the last column, we also control for the momentum factor. We still obtain a highly significant monthly alpha of 0.363% per month. Results from the four-factor Carhart model also show, that the long- and the short-side of the trading strategy contribute roughly equally to the overall performance. While the portfolio with the weakest past LTD stocks delivers an alpha of −0.188% per month, the quintile portfolio with the strongest past LTD stocks delivers an alpha of 0.175% per month (both different from zero with a significance level of 5%).

In the following, we test the robustness of our findings from Table A.14, by evaluating our trading strategy using alternative factor models. Results are presented in Table A.15.

[Insert Table A.15 about here]

First, we include the Pastor and Stambaugh (2003)'s traded liquidity risk factor in regression (1). The monthly alpha of our trading strategy is 0.365% and remains significant at the 1% significance level. In regression (2), we replace the Pastor and Stambaugh (2003) liquidity factor with the Sadka (2006) liquidity factor that is based on the permanent (variable) component of the price impact function and get similar results. In the following regressions we include the Bali, Cakici, and Whitelaw (2011a) factor to control for exposure of our strategy to lottery-type stocks and the Baker and Wurgler (2006) sentiment index orthogonoalized with respect to a set of macroeconomic conditions.[35] Then, we replace the momentum factor with the Fama-French short- and long-term reversal factors. Finally, as alternative factor models, we use the models suggested in Cremers, Petajisto, Zitzewitz (2010) that contain various combinations of return differences between the S&P 500 and Russell 2000 and 3000 subindexes as well as the momentum factor. Cremers, Petajisto, Zitzewitz (2010) propose using either four or seven factors out of the ten calculated by them.[36] In either case, we document a positive alpha of our trading strategy ranging from 0.275% up to 0.358% per month, showing that the results from our basic trading strategy are robust.

Overall, the findings from this section suggest that it is possible to also create a profitable trading strategy based on extreme dependence exposure. However, these results are only indicative, as we do not take into account any trading costs and other limits of arbitrage. Limits of arbitrage are likely to be relevant here, because we short stocks with weak LTD

[35] The lottery factor is provided by Nusret Cakici (`http://www.bnet.fordham.edu/cakici/datalibrary.htm`) and the time series of the sentiment factor is taken from `http://people.stern.nyu.edu/jwurgler/`.

[36] The data on the factor returns are taken from `http://www.petajisto.net/data.html`. The four factor model contains the factors s5rf, r2s5, r3vr3g, and the momentum factor (umd), while the seven factor model contains the factors s5rf, rms5, r2rm, s5vs5g, rmvrmg, r2vs2g, and the momentum factor (umd) (as explained in their data library).

which tend to be small and low beta stocks. Furthermore, this strategy would of course only be profitable on a risk-adjusted basis for investors who do not require a risk premium for bearing extreme dependence risk that is higher than the documented return difference.

5. Conclusion

The cross-section of expected stock returns reflects a premium for risk associated with lower tail dependence, LTD. Stocks that are characterized by strong LTD earn significantly higher average returns than stocks with weak LTD. We find that the high average returns earned by stocks with strong LTD are not explained by alternative cross-sectional effects, including market beta, size, book-to-market, momentum, liquidity, coskewness, cokurtosis, idiosyncratic volatility, and downside beta. Controlling for these and other cross-sectional effects, we find that an increase of one standard deviation in LTD is associated with an expected return premium of about 5% p.a. Our findings also suggest that most of the impact of the downside beta of Ang, Chen, and Xing (2006) seems to be driven by extreme dependence in the lower tail of the bivariate distribution of individual security and market returns.

Furthermore, we document some predictability of extreme dependence based on past extreme dependence. We can form an investable trading strategy based on past extreme dependence structures that earns an excess return over the risk free rate before trading costs of 0.338% per month, which translates into an average return premium of 4.17% p.a. over the sample period from 1963 – 2009. The Carhart (1997) four factor alpha of the strategy amounts to 4.44% p.a.

If we focus on extreme market downturns, we find that stocks with weak LTD outperform stocks with strong LTD during these periods. As stocks with weak LTD thus essentially offer an insurance against extreme negative portfolio returns, our results are consistent with the view that investors are willing to pay higher prices and eventually accept lower returns for stocks with weak LTD. The conjecture that the higher returns of stocks with strong LTD is a reflection of higher equilibrium returns in the presence of crash-averse investors is consistent with findings from the empirical literature on option prices (e.g., Rubinstein (1994)). However, the theoretical literature on crash aversion is still scarce, which offers interesting possibilities for future theoretical work.[37]

[37] Promising approaches in this direction are offered in papers on the equilibrium effect of portfolio insurance like Grossman and Zhou (1996). However, this literature typically just adds an external restriction on the lower bound of terminal portfolio wealth, i.e., those models are essentially agnostic about the motivation for portfolio insurance and crash aversion that is in a sense an assumption rather than a result.

The fact that investors earn a premium for bearing extreme dependence risk can have serious implications: If financial institutions do not have to bear the expected costs of a severe market downturn (e.g., because they expect to be bailed out in a severe crisis), they might be inclined to invest in exactly those securities that are characterized by strong lower tail dependence with the market in order to earn the associated extreme dependence premium we document.[38] Such incentives would make those institutions even more vulnerable.

[38]Some suggestive evidence along these lines is again provided in the empirical option market literature. Garleanu, Pedersen, and Poteshman (2009) document that dealers on aggregate hold short positions in out-of-the-money puts - that also offer protection against downturns - while end-users (defined as customers of brokers), seem to hold long positions, i.e., they insure against extreme downside risk.

Appendix A. Appendix: Estimating Tail Dependence Coefficients

The estimation of LTD- and UTD-coefficients can either be based on the entire set of observations or on extremal data. In the univariate setting the extreme value distributions can be expressed in parametric form (see Fisher and Tippett (1928)) and parametric extreme value theory (EVT) is the natural choice for inferences on extreme values. On the contrary, bivariate extreme value distributions (such as in this paper) cannot be characterized by a fully parametric model in general, which leads to more complicated estimation techniques. Our estimation approach relies on the entire set of observations and follows a three-step procedure.

Appendix A.1. Estimation of the Copula Parameters

We consider $j = 1, \ldots, 64$ convex combinations of copulas $C_j(\cdot, \cdot; \Theta_j)$ that each consist of one copula that allows for asymptotic dependence in the lower tail, C_{LTD}, one copula that is asymptotically independent, C_{NTD}, and one copula that allows for asymptotic dependence in the upper tail, C_{UTD} [as in (6)]. The estimation of the set of copula parameters Θ_j for the different copula combinations $C_j(\cdot, \cdot; \Theta_j)$ is performed as follows.

Let $\{r_{i,k}, r_{m,k}\}_{k=1}^{n}$ be a random sample from the bivariate distribution

$$F(r_i, r_m) = C(F_i(r_i), F_m(r_m))$$

between an individual stock return r_i and the market return r_m. We estimate the marginal distributions F_i and F_m of an individual stock return r_i and the market return r_m non-parametrically by their scaled empirical distribution functions

$$\widehat{F}_i(x) = \frac{1}{n+1} \sum_{k=1}^{n} \mathbb{1}_{r_{i,k} \leq x} \quad \text{and} \quad \widehat{F}_m(x) = \frac{1}{n+1} \sum_{k=1}^{n} \mathbb{1}_{r_{m,k} \leq x}. \tag{A.1}$$

The estimation of F_i and F_m by their empirical counterparts avoids an incorrect specification of the marginal distributions. It remains to estimate the set of copula parameters Θ_j. Since we assume a parametric form of the copula functions, the parameters Θ_j can be estimated via the maximum likelihood estimator

$$\widehat{\Theta}_j = \operatorname{argmax}_{\Theta_j} L_j(\Theta_j) \quad \text{with} \quad L_j(\Theta_j) = \sum_{k=1}^{n} \log\left(c_j(\widehat{F}_{i,r_{i,k}}, \widehat{F}_{m,r_{m,k}}; \Theta_j)\right), \tag{A.2}$$

where $L_j(\Theta_j)$ denotes the log-likelihood function and $c_j(\cdot, \cdot; \Theta_j)$ the copula densitiy. Assuming that $\{r_{i,k}, r_{m,k}\}_{k=1}^{n}$ is an i.i.d. random sample, Genest, Ghoudi, and Rivest (1995) and

Shih and Louis (1995) show that $\widehat{\Theta}$ is a consistent and asymptotic normal estimate of the set of copula parameters Θ under standard regularity conditions.[39]

Appendix A.2. How to Select the Right Copula

So far we have pointed out an estimation procedure under the assumption that the copula $C_j(\cdot,\cdot;\Theta_j)$ is known up to a set of parameters Θ_j. The choice of the copula $C^*(\cdot,\cdot;\Theta^*)$ obviously affects the resulting bivariate distribution and the resulting tail dependence coefficients LTD and UTD. However, most applications presented in the literature do not discuss this issue and rely on an arbitrary choice of the copula. To avoid this problem, we follow Ané and Kharoubi (2003) and use the empirical copula function introduced by Deheuvels (1979) and Deheuvels (1981) to evaluate the fit of different parametric copula families. We proceed as follows:

Let $\{R_{i,k}, R_{m,k}\}_{k=1}^{n}$ denote the rank statistic of $\{r_{i,k}, r_{m,k}\}_{k=1}^{n}$. Deheuvels (1979) and Deheuvels (1981) introduce the empirical copula $\widehat{C}_{(T)}$ on the lattice

$$L = \left[\left(\frac{t_1}{T}, \frac{t_2}{T}\right), t_k = 1, \ldots, n, k = 1, 2\right]$$

by the following equation:

$$\widehat{C}_{(T)}\left(\frac{t_1}{T}, \frac{t_2}{T}\right) = \frac{1}{T}\sum_{i=1}^{n} \mathbb{1}_{R_{1,i} \le t_1} \cdot \mathbb{1}_{R_{2,i} \le t_2}. \tag{A.3}$$

Following the Theorem of Sklar (1959), the bivariate empirical distribution function $\widehat{F}$ corrresponding to F is uniquely defined by the empirical marginal distributions $\widehat{F}_1$ and $\widehat{F}_2$ and the values of the empirical copula $\widehat{C}_{(T)}$ on the lattice L.

We compute Integrated Anderson-Darling distances $D_{j,IAD}$ between the parametric copulas $C_j(\cdot,\cdot;\widehat{\Theta}_j)$ and the empirical copula $\widehat{C}_{(T)}$ via

$$D_{j,IAD} = \sum_{t_1=1}^{T}\sum_{t_2=1}^{T} \frac{\left(\widehat{C}_{(T)}\left(\frac{t_1}{T}, \frac{t_2}{T}\right) - C_j\left(\frac{t_1}{T}, \frac{t_2}{T}; \widehat{\Theta}_j\right)\right)^2}{C_j\left(\frac{t_1}{T}, \frac{t_2}{T}; \widehat{\Theta}_j\right) \cdot \left(1 - C_j\left(\frac{t_1}{T}, \frac{t_2}{T}; \widehat{\Theta}_j\right)\right)}. \tag{A.4}$$

Hence, we calculate the distance between the predicted value of the parametric copulas $C_j(\cdot,\cdot;\widehat{\Theta}_j)$ and the empirical copula $\widehat{C}_{(T)}$ on every grid point on the lattice L. The estimation

[39] Obviously, daily return data often violates the assumption of an i.i.d. random sample. An alternative approach to the problem of non-i.i.d. data due to serial correlation in the first and the second moment of the time series would be to specify an ARMA-GARCH model for the univariate processes and analyze the dependence structure of the residuals. We decide not to filter our data, due to the fact that filtering will also change the data's dependence structure.

of the tail dependence coefficients LTD and UTD is based on the estimated parameters Θ^* of the copula $C^*(\cdot,\cdot;\Theta^*)$ that minimizes $D_{j,IAD}$. In unreported robustness checks, we also apply the Kolmogorov-Smirnov distances $D_{j,KS}$ between the parametric copulas $C_j(\cdot,\cdot;\widehat{\Theta}_j)$ and the empirical copula $\widehat{C}_{(T)}$, i.e.

$$D_{j,KS} = \max_{1\leq t_k\leq T} |\widehat{C}_{(T)}\left(\frac{t_1}{T},\frac{t_2}{T}\right) - C_j\left(\frac{t_1}{T},\frac{t_2}{T};\widehat{\Theta}_j\right)| \quad \text{for} \quad k=1,2, \tag{A.5}$$

as well as log-likelihood values to select the appropriate dependence structure. Independent of the selected evaluation measure ($D_{j,IAD}$, $D_{j,KS}$, or log-likelihood values), we obtain very similar results for the selected parametric copula.

Appendix A.3. Calculation of the Tail Dependence Coefficient

We compute the tail dependence coefficients implied by the estimated parameters Θ^* of the selected copula $C^*(\cdot,\cdot;\Theta^*)$. The calculation of LTD and UTD is straightforward if the copula in question has a closed form (such as copulas 1-4, A-D, and I-IV in Table A.1) and is based on formulas (4) and (5). The lower and upper tail dependence coefficient of the convex combination are calculated as the weighted sum of the LTD and UTD coefficients from the individual copulas, respectively, where the weights from (6) are used, i.e., $\text{LTD}^* = w_1^*\cdot\text{LTD}(\theta_1^*)$ and $\text{UTD}^* = (1-w_1^*-w_2^*)\cdot\text{UTD}(\theta_3^*)$.

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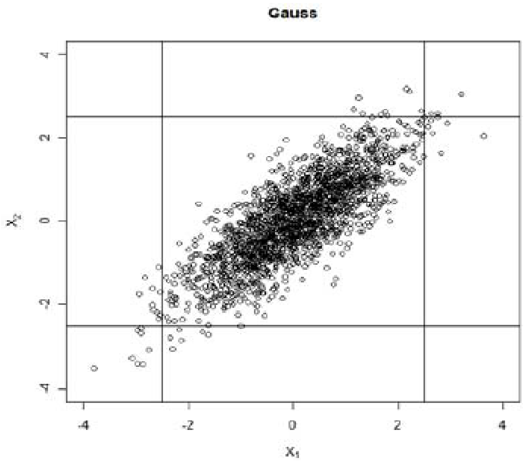
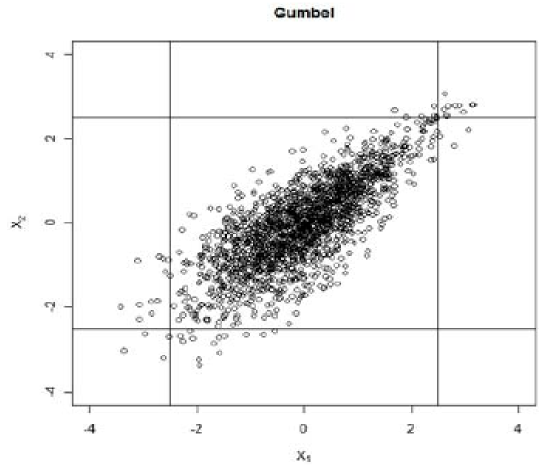
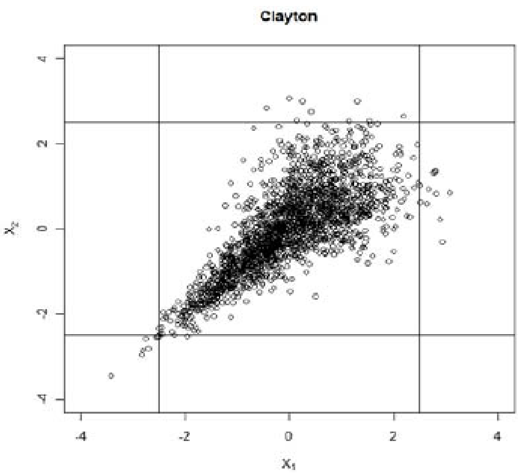
Figure A.1: Different Copula Dependence Structures



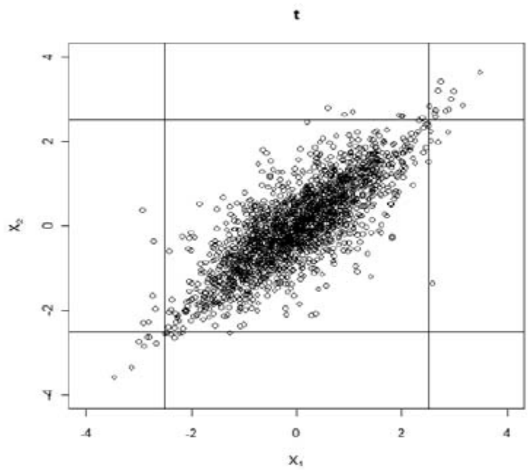
(a) Panel A: Gauss-copula structure



(b) Panel B: Gumbel-copula structure



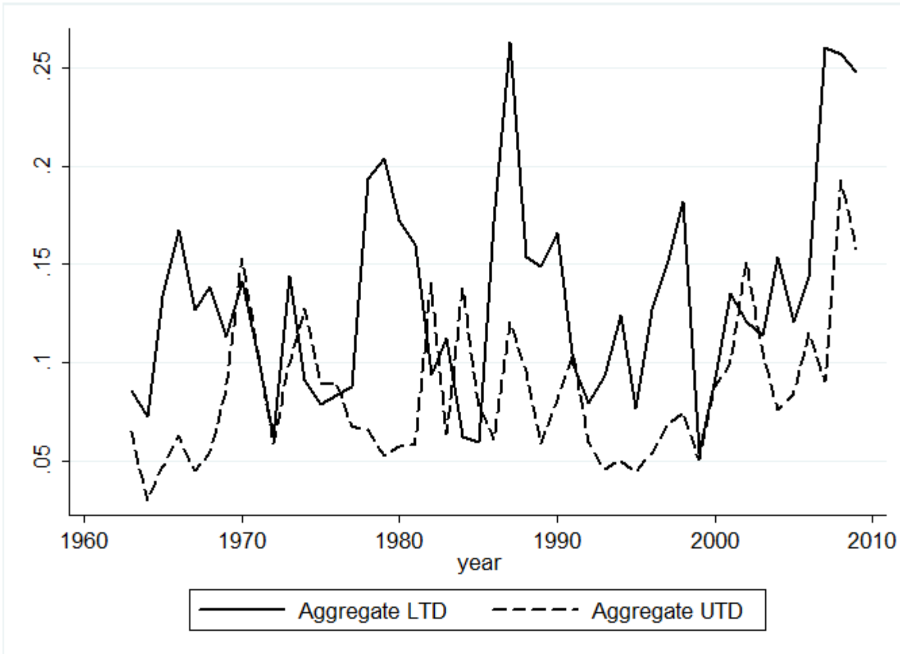
(c) Panel C: Clayton-copula structure



(d) Panel D: Student t-copula structure

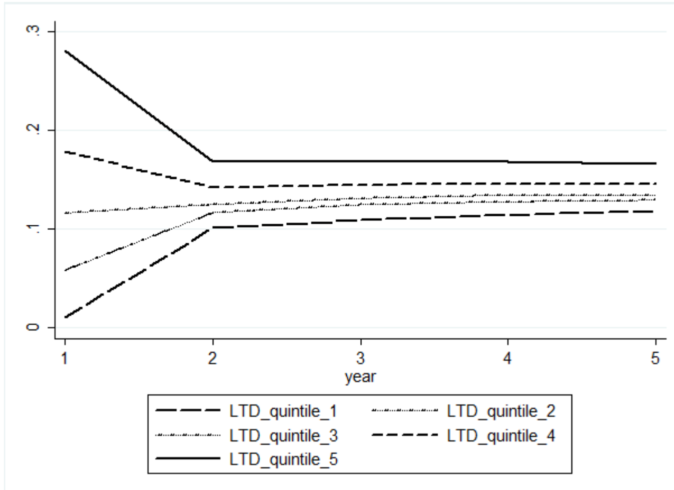
This figure displays 2,000 random variates from four bivariate distributions with standard normal marginal distributions and the Gauss-copula (Panel A), the Gumbel-copula (Panel B), the Clayton-copula (Panel C), and the Student t-copula (Panel D) determining the dependence structure. In each case, the linear correlation is set to 0.8.

Figure A.2: Aggregate Tail Dependence over Time



This figure displays the evolution of aggregate lower tail dependence, LTD, and aggregate upper tail dependence, UTD, over time. Aggregate LTD (UTD) is defined as the yearly cross-sectional, equal-weighted, average of the individual lower tail dependence coefficients, $\text{LTD}_{i,t}$ (upper tail dependence coefficients, $\text{UTD}_{i,t}$) between stock returns and market returns over all stocks i in our sample. The sample covers all U.S. common stocks traded on the NYSE / AMEX and the sample period is from January 1963 to December 2009.

Figure A.3: Persistence of LTD



This figure displays the evolution of the average equal-weighted lower tail dependence, LTD, of five quintile portfolios. Firms are sorted into quintiles based on their realized LTD between their stock return and the market return in year $t = 1$. Then, the equal-weighted average of LTD of these portfolios is computed again for the following four years. The sample covers all U.S. common stocks traded on the NYSE / AMEX and the sample period is from January 1963 to December 2009.

Table A.1: Bivariate Copula Functions with Tail Dependence Coefficients

Copula	Parametric Form	LTD	UTD
Clayton (1)	$C_{\text{Cla}}(u_1, u_2; \theta) = (u_1^{-\theta} + u_2^{-\theta} - 1)^{-1/\theta}$	$2^{-1/\theta}$	–
Rotated-Gumbel (2)	$C_{\text{RGum}}(u_1, u_2) = u_1 + u_2 - 1 + \exp\left(-((-\log(\overline{u}_1))^\theta + (-\log(\overline{u}_2))^\theta)^{1/\theta}\right)$	$2 - 2^{1/\theta}$	–
Rotated-Joe (3)	$C_{\text{RJoe}}(u_1, u_2) = u_1 + u_2 - (u_1^\theta + u_2^\theta - u_1^\theta \cdot u_2^\theta)^{1/\theta}$	$2 - 2^{1/\theta}$	–
Rotated-Galambos (4)	$C_{\text{RGal}}(u_1, u_2) = u_1 + u_2 - 1 + (\overline{u}_1) \cdot (\overline{u}_2) \cdot \exp\left(((-\log(\overline{u}_1))^{-\theta} + (-\log(\overline{u}_2))^{-\theta})^{-1/\theta}\right)$	$2^{-1/\theta}$	–
Gauss (A)	$C_{\text{Gau}}(u_1, u_2; \theta) = \Phi_\theta(\Phi^{-1}(u_1), \Phi^{-1}(u_2))$	–	–
Frank (B)	$C_{\text{Fra}}(u_1, u_2; \theta) = -\theta^{-1} \log\left(\frac{1-\exp(-\theta)-(1-\exp(-\theta u_1))(1-\exp(-\theta u_2))}{1-\exp(-\theta)}\right)$	–	–
Plackett (C)	$C_{\text{Pla}}(u_1, u_2; \theta) = \frac{1}{2}(\theta - 1)^{-1}\left\{1 + (\theta - 1)(u_1 + u_2) - [(1 + (\theta - 1)(u_1 + u_2))^2 - 4\theta u_1 u_2]^{1/2}\right\}$	–	–
F-G-M (D)	$C_{\text{Fgm}}(u_1, u_2; \theta) = u_1 u_2 (1 + \theta(1 - u_1)(\overline{u}_2))$	–	–
Joe (I)	$C_{\text{Joe}}(u_1, u_2; \theta) = 1 - ((\overline{u}_1)^\theta + (\overline{u}_2)^\theta - (\overline{u}_1)^\theta \cdot (\overline{u}_2)^\theta)^{1/\theta}$	–	$2 - 2^{1/\theta}$
Gumbel (II)	$C_{\text{Gum}}(u_1, u_2; \theta) = \exp\left(-((-\log(u_1))^\theta + (-\log(u_2))^\theta)^{1/\theta}\right)$	–	$2 - 2^{1/\theta}$
Galambos (III)	$C_{\text{Gal}}(u_1, u_2; \theta) = u_1 \cdot u_2 \cdot \exp\left(((-\log(u_1))^{-\theta} + (-\log(u_2))^{-\theta})^{-1/\theta}\right)$	–	$2^{-1/\theta}$
Rotated-Clayton (IV)	$C_{\text{RCla}}(u_1, u_2) = u_1 + u_2 - 1 + ((\overline{u}_1)^{-\theta} + (\overline{u}_2)^{-\theta} - 1)^{-1/\theta}$	–	$2^{-1/\theta}$

This table reports the parametric forms of bivariate copula functions considered in this study in the second column and the corresponding lower- and upper tail dependence coefficients, LTD and UTD, in the last two columns. The Clayton-, the Rotated Joe-, the Rotated Gumbel-, and the Rotated Galambos-copula exhibit lower tail dependence. The Gauss-, the Frank-, the Plackett-, and the FGM-copula are asymptotically independent in both tails. The Joe-, the Gumbel-, the Galambos-, and the Rotated Clayton-copula exhibit upper tail dependence. In brackets we assign a label to each basic copula. We define $\overline{u}_1 = 1 - u_1$ and $\overline{u}_2 = 1 - u_2$. Φ denotes the standard normal $N(0, 1)$ distribution function, Φ^{-1} the functional inverse of Φ and Φ_θ is the bivariate standard normal distribution function with correlation θ.

Table A.2: Frequency and Relative Percentage of Copula Selection

Copula	Freq	Perc	Copula	Freq	Perc	Copula	Freq	Perc	Copula	Freq	Perc
(1-A-I)	1,448	1.50	(2-A-I)	1,557	1.61	(3-A-I)	1,756	1.81	(4-A-I)	1,337	1.38
(1-A-II)	1,323	1.37	(2-A-II)	949	0.98	(3-A-II)	1,299	1.34	(4-A-II)	888	0.92
(1-A-III)	1,544	1.60	(2-A-III)	1,150	1.19	(3-A-III)	1,566	1.62	(4-A-III)	890	0.92
(1-A-IV)	1,596	1.65	(2-A-IV)	1,843	1.90	(3-A-IV)	1,679	1.74	(4-A-IV)	1,294	1.34
(1-B-I)	1,586	1.64	(2-B-I)	1,046	1.08	(3-B-I)	1,509	1.56	(4-B-I)	1,265	1.31
(1-B-II)	1,163	1.20	(2-B-II)	524	0.54	(3-B-II)	940	0.97	(4-B-II)	661	0.68
(1-B-III)	1,631	1.69	(2-B-III)	1,019	1.05	(3-B-III)	1,360	1.41	(4-B-III)	808	0.83
(1-B-IV)	1,486	1.54	(2-B-IV)	1,407	1.45	(3-B-IV)	1,287	1.33	(4-B-IV)	1,206	1.25
(1-C-I)	1,800	1.86	(2-C-I)	1,555	1.61	(3-C-I)	2,309	2.39	(4-C-I)	1,802	1.86
(1-C-II)	1,400	1.45	(2-C-II)	803	0.83	(3-C-II)	1,184	1.22	(4-C-II)	1,019	1.05
(1-C-III)	1,618	1.67	(2-C-III)	1,221	1.26	(3-C-III)	1,558	1.61	(4-C-III)	831	0.86
(1-C-IV)	1,359	1.40	(2-C-IV)	1,565	1.62	(3-C-IV)	1,361	1.41	(4-C-IV)	1,300	1.34
(1-D-I)	2,452	2.53	**(2-D-I)**	**2,569**	**2.65**	**(3-D-I)**	**3,180**	**3.29**	**(4-D-I)**	**2,985**	**3.08**
(1-D-II)	1,666	1.72	(2-D-II)	1,144	1.18	(3-D-II)	1,498	1.55	(4-D-II)	1,281	1.32
(1-D-III)	2,092	2.16	(2-D-III)	1,468	1.52	(3-D-III)	2,173	2.25	(4-D-III)	1,633	1.69
(1-D-IV)	2,130	2.20	(2-D-IV)	2,477	2.56	(3-D-IV)	2,020	2.09	(4-D-IV)	2,297	2.37

This table reports the absolute and percentage frequency of the selected parametric copula combinations. The appropriate dependence structure is selected by minimizing the distance between the parametric copulas and the empirical copula via the Integrated Anderson-Darling distance. In columns 1, 4, 7, and 10 we indicate the label of the respective copula combination based on the basic copula labels from Table A.1. The three copulas that are most often selected are: (3-D-I) - Rotated-Joe/F-G-M/Joe-copula, (4-D-I) - Rotated-Galambos/F-G-M/Joe-copula, and (2-D-I) - Rotated-Gumbel/F-G-M/Joe-copula (these copula combinations are marked in bold).

Table A.3: Aggregate Tail Dependence Over Time

Time Period	Obs	Aggr. LTD	Aggr. UTD	Difference	Aggr. LTD Large Firms	Aggr. LTD Small Firms	Difference
1963 – 1968	12021	0.122	0.051	0.071***	0.146	0.073	0.073***
1969 – 1974	13939	0.109	0.105	0.004***	0.123	0.081	0.042***
1975 – 1979	11275	0.129	0.073	0.056***	0.153	0.078	0.079***
1980 – 1984	10599	0.121	0.091	0.030***	0.135	0.086	0.049**
1985 – 1989	9726	0.159	0.083	0.076***	0.261	0.084	0.177***
1990 – 1994	9640	0.112	0.067	0.045***	0.161	0.067	0.094***
1995 – 1999	11415	0.120	0.059	0.061***	0.182	0.065	0.127***
2000 – 2004	9483	0.122	0.104	0.018***	0.166	0.065	0.101***
2005 – 2009	8669	0.203	0.126	0.077***	0.254	0.099	0.155***
1963 – 2009	96767	0.131	0.083	0.048***	0.176	0.078	0.098***

This table reports equal-weighted average lower (upper) tail dependence coefficients, LTD (UTD), between individual stock returns and market returns for 5-year subsamples and over the whole sample period from January 1963 to December 2009 in Columns 3 and 4. The difference between the average LTD and UTD coefficients is displayed in column 5. In Columns 6 and 7 we compute average LTD coefficients for the 20% and the 20% smallest firms according to their market capitalization and column 8 reports the difference. The sample covers all U.S. common stocks traded on the NYSE / AMEX. ***, **, and * indicate significance at the one, five, and ten percent level, respectively.

Table A.4: Correlations

	LTD	UTD	β	β^-	β^+	size	bookmarket	illiq	past return	idio vola	coskew	cokurt	max
LTD	1.00	-	-	-	-	-	-	-	-	-	-	-	-
UTD	0.12	1.00	-	-	-	-	-	-	-	-	-	-	-
β	0.38	0.31	1.00	-	-	-	-	-	-	-	-	-	-
β^-	0.49	0.07	0.77	1.00	-	-	-	-	-	-	-	-	-
β^+	0.19	0.48	0.78	0.47	1.00	-	-	-	-	-	-	-	-
size	0.29	0.30	0.05	0.03	0.16	1.00	-	-	-	-	-	-	-
bookmarket	−0.09	−0.11	−0.11	−0.07	−0.08	−0.34	1.00	-	-	-	-	-	-
illiq	−0.28	−0.28	−0.22	−0.08	−0.19	−0.84	0.30	1.00	-	-	-	-	-
past return	0.08	−0.05	0.10	0.12	0.05	0.07	0.18	−0.01	1.00	-	-	-	-
idio vola	−0.09	−0.09	0.23	0.27	0.12	−0.42	0.05	0.31	−0.13	1.00	-	-	-
coskew	−0.37	0.23	0.07	−0.12	0.24	−0.05	0.01	0.07	−0.09	0.03	1.00	-	-
cokurt	0.38	0.23	0.21	0.16	0.20	0.21	−0.09	−0.21	−0.00	−0.06	−0.69	1.00	-
max	−0.08	−0.10	0.14	0.18	0.07	−0.39	0.14	0.31	0.04	0.59	0.03	−0.08	1.00

This table displays linear correlations between the independent variables used in this study. As independent variables we use lower tail dependence (LTD), upper tail dependence (UTD), beta (β), downside beta (β^-), upside beta (β^+), the log of market capitalization (size), book-to-market value (bookmarket), illiquidity (illiq), past return, idiosyncratic volatility (idio vola), coskewness (coskew), cokurtosis (cokurt), and the maximum daily return over the past one year (max). A detailed description of the computation of these variables is given in the main text. The sample covers all U.S. common stocks traded on the NYSE / AMEX and the sample period is from January 1963 to December 2009.

Table A.5: Univariate Equal-weighted Portfolio Sorts: Tail Dependence and Returns

Panel A: Lower Tail Dependence (LTD)

Portfolio	Return	LTD	UTD	β	β^-	coskew	size	bookmarket
1 Weak LTD	3.99%	0.01	0.06	0.56	0.43	0.00	11.35	0.98
2	8.84%	0.06	0.08	0.73	0.77	−0.05	11.79	0.92
3	10.39%	0.12	0.09	0.82	0.96	−0.09	12.01	0.90
4	14.07%	0.18	0.10	0.95	1.18	−0.12	12.36	0.86
5 Strong LTD	19.70%	0.29	0.09	1.19	1.54	−0.17	12.97	0.82
Strong - Weak	15.71%***	0.28***	0.03***	0.63***	1.11***	−0.17***	1.62***	−0.16***

Panel B: Upper Tail Dependence (UTD)

Portfolio	Return	UTD	LTD	β	β^+	coskew	size	bookmarket
1 Weak UTD	13.87%	0.00	0.11	0.69	0.34	−0.15	11.59	0.95
2	13.44%	0.01	0.12	0.69	0.41	−0.13	11.52	0.96
3	11.45%	0.06	0.14	0.82	0.72	−0.09	11.93	0.92
4	10.51%	0.12	0.15	0.93	0.98	−0.06	12.31	0.88
5 Strong UTD	7.85%	0.23	0.14	1.11	1.35	−0.01	13.10	0.79
Strong - Weak	−6.02%***	0.23***	0.03***	0.42***	1.01***	0.14***	1.51***	−0.16***

This table reports equal-weighted average returns and risk characteristics of stocks sorted by realized LTD (Panel A) and UTD (Panel B). In each year, we rank stocks into quintiles (1-5) and form equal-weighted portfolios at the beginning of each annual period. The column labelled 'Return' reports the average return in excess of the one-month T-bill rate over the year. The other columns report average risk and firm characteristics as described in Table A.4 and measured contemporaneously with returns. The row labelled 'Strong-Weak' reports the difference between the returns of portfolio 5 and portfolio 1 with corresponding statistical significance levels. The sample covers all U.S. common stocks traded on the NYSE/AMEX and the sample period is from January 1963 to December 2009. ***, **, and * indicate significance at the one, five, and ten percent level, respectively.

Table A.6: Excess Returns of LTD-Portfolios during Financial Crises

Portfolio	Black Monday	Asia Crisis	Dot-Com Bubble Burst	Lehman Crisis	$r_{m,d} < -5\%$
1 Weak LTD	−9.5%	−2.4%	−1.7%	−5.9%	−4.4%
2	−13.3%	−4.4%	−3.1%	−6.9%	−6.0%
3	−15.7%	−5.7%	−4.3%	−9.4%	−7.3%
4	−16.3%	−6.3%	−5.9%	−11.2%	−8.4%
5 Strong LTD	−18.7%	−6.8%	−7.3%	−11.8%	−9.2%
Strong - Weak	−9.2%	−4.4%	−5.6%	−5.9%	−4.8%***

This table reports equal-weighted daily excess returns returns of stocks sorted by realized LTD. Each year we rank stocks into quintiles (1-5) and form equal-weighted portfolios at the beginning of each annual period. We investigate daily excess returns of these portfolios during "Black Monday" (October 19, 1987), the Asian Crisis (October 27, 1997), the burst of the dot-com bubble (April 14, 2000), and the recent Lehman crises (October 15, 2008). The last column reports average daily returns in excess of the one-month T-bill rate of the portfolios when the daily excess market return, $r_{m,d}$, on day d was below −5%. The row labelled 'Strong-Weak' reports the difference between the returns of portfolio 5 and portfolio 1 with corresponding statistic significance level (only last column). The sample covers all U.S. common stocks traded on the NYSE / AMEX. ***, **, and * indicate significance at the one, five, and ten percent level, respectively.

Table A.7: Dependent Portfolio Sorts

Panel A: Beta (β) and Lower Tail Dependence (LTD)

Portfolio	1 Low β	2	3	4	5 High β	Average
1 Weak LTD	4.48%	3.57%	4.53%	5.97%	10.40%	5.79%
2	6.60%	7.67%	8.81%	12.64%	14.28%	10.00%
3	6.48%	9.47%	9.29%	13.53%	20.32%	11.82%
4	6.95%	9.05%	12.78%	14.29%	23.92%	13.40%
5 Strong LTD	9.71%	12.04%	13.76%	16.76%	28.69%	16.19%
Strong - Weak	5.23%***	8.48%***	9.23%***	10.79%***	18.29%***	10.40%***

Panel B: Downside Beta (β^-) and Lower Tail Dependence (LTD)

Portfolio	1 Low β^-	2	3	4	5 High β^-	Average
1 Weak LTD	3.12%	4.86%	4.58%	7.88%	15.01%	7.09%
2	3.07%	6.79%	8.30%	11.71%	17.76%	9.52%
3	6.18%	7.95%	9.48%	12.26%	21.97%	11.57%
4	7.60%	8.65%	12.38%	13.99%	23.79%	13.28%
5 Strong LTD	10.01%	10.02%	12.56%	17.27%	27.99%	15.57%
Strong - Weak	6.89%***	5.16%***	7.98%***	9.40%***	12.98%***	8.48%***

Panel C: Coskewness (coskew) and Lower Tail Dependence (LTD)

Portfolio	1 Low coskew	2	3	4	5 High coskew	Average
1 Weak LTD	10.68%	7.49%	5.18%	2.60%	3.13%	5.81%
2	12.58%	11.15%	10.03%	6.65%	4.46%	8.97%
3	15.35%	12.08%	9.91%	7.97%	6.88%	10.44%
4	18.94%	15.41%	13.10%	12.42%	8.70%	13.71%
5 Strong LTD	23.58%	19.94%	18.06%	16.32%	12.87%	18.16%
Strong - Weak	12.90%***	12.46%***	12.88%***	13.73%***	9.74%***	12.34%***

This table reports equal-weighted average return and risk characteristics of 25 portfolios double-sorted on realized LTD and realized beta (Panel A), realized downside beta (Panel B), and realized coskewness (Panel C), respectively. First, we form quintile portfolios sorted on beta, downside beta, and coskewness, respectively. Then, within each of those quintiles, we sort stocks into five equal-weighted portfolios based on LTD. The row labelled 'Strong - Weak' reports the difference between the returns of portfolio 5 and portfolio 1 in each beta, downside beta, or coskewness quintile with corresponding statistic significance level. The sample covers all U.S. common stocks traded on NYSE / AMEX and the sample period is from January 1963 to December 2009. ***, **, and * indicate significance at the one, five, and ten percent level, respectively.

Table A.8: Fama-MacBeth (FMB) Regressions

	(1) return	(2) return	(3) return	(4) return	(5) return	(6) return	(7) return	(8) return	Econ. Signif.
β	0.0907***			0.103***	0.0748**	0.140***	0.173***	0.169***	9.45%
	(3.51)			(3.63)	(2.64)	(5.98)	(5.65)	(5.37)	
β^-		0.0815***					-0.00132	-0.00413	0.30%
		(3.74)					(-0.11)	(-0.35)	
β^+		-0.0107					-0.0380***	-0.0374***	-2.89%
		(-1.36)					(-2.90)	(-2.86)	
LTD			0.584***		0.555***	0.452***	0.472***	0.465***	5.28%
			(9.11)		(11.59)	(9.88)	(10.31)	(10.01)	
UTD			-0.326***		-0.254***	-0.299***	-0.270***	-0.276***	-2.59%
			(-5.00)		(-4.45)	(-7.14)	(-5.87)	(-6.04)	
size				-0.00738	-0.0121*	-0.0274***	-0.0273***	-0.0304***	-6.44%
				(-1.00)	(-1.70)	(-5.52)	(-5.44)	(-5.87)	
bookmarket				0.0415***	0.0383***	0.0306***	0.0314***	0.0307***	2.24%
				(4.72)	(4.50)	(3.95)	(4.09)	(4.04)	
coskew				-0.116**	0.127**	0.0910**	0.168***	0.159***	4.88%
				(-2.62)	(2.63)	(2.22)	(4.01)	(3.56)	
illiq				0.221***	0.228***	0.205***	0.214***	0.206***	2.73%
				(3.99)	(4.20)	(3.89)	(4.11)	(3.84)	
past return						-0.0257	-0.0273	-0.0188	-0.86%
						(-1.21)	(-1.27)	(-0.88)	
idio vola						-4.638***	-4.471***	-3.522*	-4.17%
						(-2.99)	(-2.71)	(-1.97)	
cokurt								0.0133	3.55%
								(1.17)	
max								-0.246**	-1.78%
								(-2.52)	
constant	0.0369*	0.0391*	0.0678*	0.0102	0.0558	0.347***	0.336***	0.375***	
	(1.69)	(1.76)	(1.92)	(0.10)	(0.56)	(4.47)	(4.38)	(4.95)	
R^2	0.034	0.040	0.024	0.094	0.110	0.143	0.149	0.155	

This table displays the results of Fama-MacBeth (1973) regressions of 1-year excess returns over the risk free rate on beta (β), downside beta (β^-) and upside beta (β^+), LTD and UTD, the log of market capitalization (size), book-to-market ratio (bookmarket), coskewness (coskew), the Amihud Illiquidity Ratio (illiq), the past 12-month excess returns (past return), idiosyncratic volatility (idio vola), cokurtosis (cokurt), and the maximum daily return over the past one year (*max*). All risk characteristics are calculated contemporaneously to the yearly excess return. The independent variables are winsorized at the 1% level and at the 99% level. The last column displays the change in annualized excess returns for a one standard deviation increase in the respective independence variable based on regression (8). The sample covers all U.S. common stocks traded on the NYSE / AMEX and the sample period is from January 1963 to December 2009. t-statistics are in parentheses. ***, **, and * indicate significance at the one, five, and ten percent level, respectively.

Table A.9: FMB with Fixed Copula Combinations & Industry- and DGTW-adjusted Returns

Fixed Copula	LTD (t-stat)	R^2	Return Adjustment	LTD (t-stat)	R^2
(3-D-I)	0.591***	0.168	FF-12	0.446***	0.132
	(9.95)			(9.37)	
(4-D-I)	0.492***	0.169	FF-48	0.413***	0.114
	(7.44)			(9.58)	
(2-D-I)	0.477***	0.163	SIC-2	0.414***	0.114
	(7.06)			(9.94)	
(2-B-II)	0.567***	0.162	SIC-3	0.358***	0.093
	(10.17)			(9.32)	
(4-B-II)	0.487***	0.166	SIC-4	0.294***	0.077
	(10.15)			(8.73)	
(4-B-III)	0.539***	0.167	DGTW	0.464***	0.091
	(9.93)			(9.32)	

This table shows results for the estimate of the influence of LTD from Fama-MacBeth (1973) regressions of 1-year excess returns on LTD on the full set of controls as in Regression (5) from Table A.8 (included in the regression but suppressed in the table) in the first three columns. LTD coefficients are calculated based on the Rotated Joe/F-G-M/Joe copula (3-D-I), the Rotated Galambos/F-G-M/Joe copula (4-D-I), the Rotated Gumbel/F-G-M/Joe copula (2-D-I), the Rotated Joe/Frank/Gumbel copula (2-B-II), the Rotated Galambos/Frank/Gumbel copula (4-B-II), and the Rotated Galambos/Frank/Galambos copula (4-B-III). The last three columns repeat the same Fama-MacBeth (1973) regressions with the full set of controls as in Regression (8) from Table A.8 with alternative dependent variables. We adjust returns by industry (based on Kenneth French's 12 (FF-12) and 48 (FF-48) industry portfolios as well as SIC 2-, 3-, and 4-digit codes. In the last line, we report the regression results when we adjust returns for DGTW characteristic-based benchmarks. The sample covers all U.S. common stocks traded on the NYSE / AMEX and the sample period is from January 1963 to December 2009. t-statistics are in parentheses. ***, **, and * indicate significance at the one, five, and ten percent level, respectively.

Table A.10: Univariate Equal-weighted Portfolio Sorts: Tail Dependence and Alphas

Portfolio	CAPM-Alpha	FF-Alpha	CAR-Alpha
1 Weak LTD	1.33%	−2.55%	−1.45%
2	4.61%	0.33%	1.00%
3	5.67%	1.14%	1.41%
4	8.56%	3.55%	3.71%
5 Strong LTD	12.59%	7.55%	6.74%
Strong - Weak	11.26%***	10.10%***	8.19%***

This table reports equal-weighted average alphas of stocks sorted by realized LTD. Each year we rank stocks into quintiles (1-5) and form equal-weighted portfolios at the beginning of each annual period. The column labelled 'CAPM-Alpha' reports the average yearly alpha with regard to Sharpe (1964)'s capital asset pricing model. The column labelled 'FF-Alpha' reports average yearly alpha with regard to Fama and French (1993)'s three factor model. Finally, in the column labelled 'CAR-Alpha', we report the average yearly alpha with regard to Carhart (1997)'s four factor model. The row labelled 'Strong - Weak' reports the difference between the alphas of portfolio 5 (High LTD) and portfolio 1 (Low LTD) with corresponding statistic significance level. The sample covers all U.S. common stocks traded on the NYSE / AMEX and the sample period is from January 1963 to December 2009. ***, **, and * indicate significance at the one, five, and ten percent level, respectively.

Table A.11: Value-weighted Portfolio Sorts

Panel A: Univariate Sorts on Lower Tail Dependence (LTD)

Portfolio	1 Weak LTD	2	3	4	5 Strong LTD	Strong - Weak
Return	−1.03%	2.23%	3.73%	5.53%	9.45%	10.48%***

Panel B: Double-sorts on Beta (β) and Lower Tail Dependence (LTD)

Portfolio	1 Low β	2	3	4	5 High β	Average
1 Weak LTD	3.68%	1.81%	−0.05%	−0.59%	0.01%	0.97%
2	1.80%	3.81%	3.56%	4.65%	3.40%	3.44%
3	3.85%	5.77%	3.04%	4.34%	7.03%	4.80%
4	5.67%	6.19%	6.52%	5.07%	9.49%	6.59%
5 Strong LTD	6.44%	6.85%	7.90%	9.28%	16.23%	9.34%
Strong - Weak	2.75%**	5.04%***	7.95%***	9.87%***	16.22%***	8.37%***

Panel C: Double-sorts on Downside Beta (β^-) and Lower Tail Dependence (LTD)

Portfolio	1 Low β	2	3	4	5 High β	Average
1 Weak LTD	3.59%	−0.22%	1.38%	0.54%	1.47%	1.35%
2	1.43%	3.13%	3.99%	5.31%	5.52%	3.88%
3	2.39%	4.94%	4.80%	4.57%	8.94%	5.13%
4	5.65%	5.02%	6.91%	7.46%	10.42%	7.09%
5 Strong LTD	6.11%	6.86%	7.00%	11.59%	15.46%	9.40%
Strong - Weak	2.52%	7.07%***	5.62%***	11.05%***	13.99%***	8.05%***

Panel D: Double-sorts on Coskewness (coskew) and Lower Tail Dependence (LTD)

Portfolio	1 Low β	2	3	4	5 High β	Average
1 Weak LTD	0.78%	0.84%	−0.95%	−1.81%	0.09%	−0.21%
2	3.99%	2.20%	0.59%	2.22%	1.08%	2.02%
3	4.64%	6.27%	4.83%	1.12%	3.09%	3.99%
4	7.50%	5.09%	5.38%	3.88%	3.35%	5.04%
5 Strong LTD	11.80%	10.27%	7.63%	8.70%	6.98%	9.08%
Strong - Weak	11.02%***	9.42%***	8.58%***	10.51%***	6.89%***	9.29%***

Table A.11: continued

This table reports value weighted returns from various sorts. In Panel A, all stocks are sorted by realized LTD. Each year we rank stocks into quintiles (1-5) and form value-weighted portfolios at the beginning of each annual period. The row labelled 'Return' reports the average return in excess of the one-month T-bill rate over the next year in the respective quintile. The following Panels report value-weighted average returns of 25 portfolios sorted by realized beta and realized LTD (Panel B), realized downside beta and realized LTD (Panel C), and realized coskewness and realized LTD (Panel D). First, we form quintile portfolios sorted on beta, downside beta, and coskewness, respectively. Then, within each beta, downside beta, and coskewness quintile, we sort stocks into five portfolios based on LTD. In Panel A (Panels B to D) the column (row) labelled 'Strong - Weak' reports the difference between the returns of portfolio 5 and portfolio 1 with corresponding statistic significance level. The sample covers all U.S. common stocks traded on the NYSE / AMEX and the sample period is from January 1963 to December 2009. ***, **, and * indicate significance at the one, five, and ten percent level, respectively.

Table A.12: Temporal Stability: Portfolio Sorts and Fama-MacBeth (1973) Regressions

Panel A: Univariate Sorts

Portfolio	Jan 63 - Dec 86	Jan 87 - Dec 09
1 Weak LTD	4.67%	3.27%
2	8.98%	8.69%
3	9.80%	11.00%
4	14.54%	13.57%
5 Strong LTD	20.91%	18.44%
5 Strong - Weak	16.24%***	15.17%***

Panel B: Bivariate Sorts

	Beta		Down Beta		Coskew	
Portfolio	63 - 86	87-09	63 - 86	87-09	63 - 86	87-09
1 Weak LTD	7.64%	3.86%	8.97%	5.13%	6.73%	4.86%
2	10.10%	9.89%	10.25%	8.77%	8.20%	9.78%
3	11.89%	11.74%	10.80%	12.36%	10.47%	10.40%
4	13.29%	13.50%	13.09%	13.48%	14.19%	13.22%
5 Strong LTD	16.08%	16.31%	15.74%	15.39%	19.44%	16.82%
5 Strong - Weak	8.45%***	12.45%***	6.77%***	10.26%***	12.71%***	11.95%***

Table A.12: Continued

Panel C: Fama-MacBeth (1973) Regressions

	Jan1963-Dec1972	Jan1973-Dec1981	Jan1982-Dec1990	Jan1991-Dec1999	Jan2000-Dec2009	Up Market	Down Market
β	0.121**	0.176**	0.121	0.308***	0.125	0.227***	0.0366
	(2.44)	(2.48)	(1.80)	(3.99)	(1.63)	(5.99)	(0.95)
β^-	0.0122	-0.0305	0.0135	0.0224	-0.0349	-0.00913	0.00729
	(0.42)	(-1.60)	(0.62)	(0.68)	(-1.24)	(-0.58)	(0.43)
β^+	-0.00165	-0.0389***	-0.0357	-0.0659	-0.0442	-0.0469**	-0.0158
	(-0.05)	(-4.64)	(-1.39)	(-1.51)	(-1.61)	(-2.64)	(-1.21)
LTD	0.387**	0.489***	0.476***	0.627***	0.358***	0.560***	0.249***
	(3.05)	(4.80)	(3.84)	(7.50)	(4.64)	(11.11)	(3.35)
UTD	-0.333*	-0.266***	-0.318***	-0.111	-0.343***	-0.288***	-0.247***
	(-1.87)	(-4.43)	(-3.76)	(-1.27)	(-5.36)	(-4.61)	(-5.15)
size	-0.0233*	-0.0332**	-0.0290*	-0.0432**	-0.0241**	-0.0416***	-0.00496
	(-2.20)	(-2.76)	(-2.28)	(-3.10)	(-2.41)	(-7.04)	(-0.75)
bookmarket	0.0176	0.0456**	0.0361**	0.0328*	0.0223*	0.0320***	0.0277
	(0.69)	(2.37)	(2.41)	(2.19)	(2.18)	(3.76)	(1.72)
coskew	0.0168	0.154*	0.245***	0.166	0.209***	0.174***	0.126**
	(0.14)	(1.92)	(3.93)	(1.05)	(3.56)	(2.90)	(2.30)
illiq	0.137	0.259***	0.239*	0.287***	0.117	0.206***	0.205**
	(1.00)	(3.56)	(2.14)	(4.58)	(0.66)	(2.98)	(2.51)
past return	-0.0147	0.00462	0.0291	0.00152	-0.105	-0.0247	-0.00550
	(-0.52)	(0.12)	(1.09)	(0.07)	(-1.29)	(-0.82)	(-0.31)
idio vola	4.363	-0.750	-9.148***	-7.858***	-4.146**	-0.878	-9.566***
	(0.54)	(-0.33)	(-6.28)	(-7.26)	(-2.37)	(-0.37)	(-7.70)
cokurt	-0.00629	0.0433	0.0281	-0.00481	0.00691	0.0122	0.0159
	(-0.22)	(1.39)	(1.34)	(-0.21)	(0.29)	(0.87)	(0.79)
max	-0.817**	-0.468**	-0.238***	0.0788	0.168	-0.369***	0.0359
	(-2.83)	(-2.51)	(-3.96)	(0.47)	(0.94)	(-3.04)	(0.26)
constant	0.190	0.259	0.460**	0.565**	0.399**	0.518***	0.0474
	(0.97)	(1.53)	(3.34)	(3.08)	(2.41)	(5.81)	(0.47)
R^2	0.227	0.162	0.149	0.106	0.134	0.145	0.177

Panel A of this table lists equal-weighted average returns of stocks sorted by realized LTD in the two time periods from January 1963 to December 1986 and from January 1987 to December 2009. Each year we rank stocks into quintiles (1-5) and form equal-weighted portfolios at the beginning of each annual period. We report the average return in excess of the one-month T-bill rate over the next year. The row labelled 'Strong - Weak' reports the difference between the returns of portfolio 5 and portfolio 1 with corresponding statistic significance level. In Panel B, we report the results from the same double sorts as in Table A.7 separately for the pre- and post-1987 period. We only report the average return across the beta, downside-beta, and coskewness, respectively, portfolios for the 5 LTD portfolios as well as the difference portfolio. Panel C displays the results of Fama and MacBeth (1973) regressions of 1-year excess returns on firm characteristics and realized risk characteristics for, respectively, 9- and 10 year subsamples. We include the full set of independent variables from Regression (8) in Table A.8. All independent variables are winsorized at the 1% level and at the 99%. The sample covers all U.S. common stocks traded on the NYSE / AMEX and the respective sample period is displayed in the first row. t statistics are in parentheses. ***, **, and * indicate significance at the one, five, and ten percent level, respectively.

Table A.13: Different Regression Methods

Regression	(1)	(2)	(3)	(4)	(5)
LTD	0.474***	0.460***	0.460***	0.431***	0.461***
	(9.84)	(22.65)	(18.14)	(20.95)	(22.70)
Controls	yes	yes	yes	yes	yes
Method	fmb	ols	ols	panel	panel
Windsorized	no	yes	yes	yes	yes
Year Effects		yes	yes	yes	yes
Firm Effects		no	no	fixed	random
Clustered SE		firm	industry	no	no
R^2	0.152	0.227	0.227	0.282	

This table reports the results regressions of excess returns on firm- and risk characteristics from using various regression techniques. The independent variables are the same as in Regression (8) in Table A.8. We only display the results for the coefficient estimate for the impact of LTD. All other variables are included in the regression but suppressed in the table. Regression (1) performs a Fama-Macbeth-Regression, but we do not winsorize the independent variables. In Regression (2) we perform a pooled OLS-regression with time-fixed effects and standard errors clustered by single stocks. Regression (3) is identical, but we cluster standard errors by Fama-French 12 industry. Regression (4) performs a panel data regression with firm fixed effects. In Regression (5) we regress excess returns on the independent variables via a random-effect panel data regression. The sample covers all U.S. common stocks traded on the NYSE / AMEX and the sample period is from January 1963 to December 2009. t-statistics are in parentheses. ***, **, and * indicate significance at the one, five, and ten percent level, respectively.

Table A.14: Past Lower Tail Dependence and Future Returns

Portfolio	Return	CAPM-Alpha	FF-Alpha	CAR-Alpha
1 Weak LTD	0.509%	0.106%	$-0.324\%^{***}$	$-0.188\%^{**}$
2	0.672%	$0.237\%^{*}$	$-0.189\%^{**}$	-0.024%
3	0.709%	$0.251\%^{**}$	$-0.134\%^{*}$	$+0.016\%$
4	0.776%	$0.295\%^{**}$	-0.038%	$+0.108\%^{*}$
5 Strong LTD	0.850%	$0.339\%^{***}$	$+0.110\%$	$+0.175\%^{**}$
5 Strong - Weak	$0.341\%^{***}$	$0.233\%^{**}$	$0.434\%^{***}$	$0.363\%^{***}$

This table lists equal-weighted average returns and risk characteristics of stocks sorted by past lower tail dependence (LTD). Each month we rank stocks into quintiles (1-5) and form equal-weighted portfolios based on past 1-year LTD. The column labelled 'Return' reports the average return in excess of the one-month T-bill rate over the following month. The other columns report average realized risk characteristics measured contemporaneously with returns. The row labelled 'Strong - Weak' reports the difference between the returns of portfolio 5 and portfolio 1 with corresponding statistic significance level. The sample period is from January 1963 to December 2009. ***, **, and * indicate significance at the one, five, and ten percent level, respectively.

Table A.15: Trading Strategy Based on Lower Tail Dependence: Factor Models

	(1) trad strat	(2) trad strat	(3) trad strat	(4) trad strat	(5) trad strat	(6) trad strat	(7) trad strat
marketrf	0.279***	0.230***	0.306***	0.305***	0.299***		
	(11.94)	(7.38)	(12.21)	(12.85)	(13.06)		
smb	-0.413***	-0.453***	-0.397***	-0.397***	-0.307***		
	(-13.35)	(-11.00)	(-10.44)	(-12.38)	(-9.17)		
hml	-0.296***	-0.222***	-0.237***	-0.242***	-0.150***		
	(-8.28)	(-4.58)	(-6.14)	(-6.73)	(-3.94)		
umd	0.101***	0.0504*	0.0763***	0.0737***		0.0692**	0.0121
	(4.31)	(1.75)	(3.32)	(3.24)		(2.56)	(0.39)
ps liqui	-0.0000323						
	(-0.98)						
sadka liqui		-1.390**					
		(-2.45)					
max ew			0.00455				
			(0.16)				
sent orth				-0.000957			
				(-0.98)			
rev short					-0.0776**		
					(-2.57)		
rev long					-0.262***		
					(-6.00)		
s5rf						0.328***	0.270***
						(10.95)	(7.69)
r2s5						-0.279***	
						(-7.34)	
r3vr3g						-0.105**	
						(-2.19)	
rms5							-0.0174
							(-0.19)
r2rm							-0.525***
							(-6.72)
s5vs5g							-0.0569
							(-0.69)
rmvrmg							-0.0568
							(-0.57)
r2vr2g							0.00186
							(0.02)
alpha	0.365%***	0.532%***	0.365%***	0.378%***	0.490%***	0.275%**	0.358%**
	(3.57)	(4.17)	(3.65)	(3.69)	(5.14)	(2.18)	(2.47)
R^2	0.426	0.410	0.396	0.399	0.428	0.352	0.350

This table lists OLS-regression results of a trading strategy based on the difference of high past LTD (quintile 5) and low past LTD (quintile 1) on alternative factor models. Regression (1) consists of the Carhart (1997) four-factor model plus the Pastor and Stambaugh (2003)'s traded liquidity risk factor. In regression (2), we replace the Pastor and Stambaugh (2003) liquidity factor by the Sadka (2006) liquidity factor. In regressions (3)-(4) we include the Bali, Cakici, and Whitelaw (2011a) lottery factor and the Baker and Wurgler (2006) sentiment index orthogonoalized with respect to a set of macroeconomic conditions. In regression (5), we replace the momentum factor by the Fama-French short- and long-term reversal factors. Finally, as alternative factor models in regressions (6)-(7), we use the models suggested in Cremers, Petajisto, Zitzewitz (2010) that contain various combinations of return differences between the S&P 500 and Russell 2000 and 3000 subindexes as well as the momentum factor. Portfolios are rebalanced monthly. The sample period is from January 1963 to December 2009. ***, **, and * indicate significance at the one, five, and ten percent level, respectively.